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THE HISTORY OF IRRATIONAL NUMBERS

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Number is one of the fundamental concepts in mathematics and has emerged from practical human needs. The origin and development of numbers can be described in the following early stages:

Natural numbers arose from the need to measure and distribute quantities.

Positive numbers were created due to the needs of mathematics itself, namely, to solve and justify algebraic equations. Zero appeared as a result of introducing negative numbers. This list could be extended further, but we will now turn to the history of irrational numbers, which appeared after the aforementioned types of numbers.

In the Pythagorean school (5th century BC), it was proven that rational numbers are not sufficient to precisely measure all line segments; there exist segments that are incommensurable. For instance, the side of a square with area 2 is not commensurable with its diagonal. This is proven through contradiction in Euclid’s "Elements".

This discovery contradicted Pythagorean doctrine, which held that any quantity could be expressed through whole numbers and their ratios. Initially, they attempted to keep this discovery secret.

Hippasus of Metapontum (5th century BC) continued this work, and by the end of the same century, Theodorus of Cyrene demonstrated that the sides of squares with areas 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, and 17 are not commensurable with the side of a unit square—i.e., they are irrational. Theaetetus generalized this idea by proving the irrationality of $\sqrt{N}$ for any whole number N that is not a perfect square. Realizing that infinitely many segments and geometric quantities cannot be measured using whole or fractional numbers, the Pythagoreans attempted to base geometry and algebra not on numbers but on geometry itself. Thus, geometric algebra was created and developed. Based on this, mathematicians began to represent whole numbers and any quantity using line segments, rectangles, and other geometric shapes.

In the Arab East, mathematics began to develop from the 7th century onwards. In this period, many Central Asian scholars made significant discoveries related to the concept of numbers, such as: Al-Khwarizmi (783–850), Abu Rayhan Biruni(973–1048), Avicenna (Ibn Sina) (980–1037), Abu Nasr al-Farabi (873–950), Omar Khayyam (1048–1131), Some of their contributions include:

1. Development of methods for extracting square roots from numbers
2. Discovery of decimal fractions
3. Expansion of the concept of positive real numbers

Although Al-Khwarizmi, in his work On the Calculation with Hindu Numerals, provided a detailed explanation of the decimal system, it only started being widely used 300 years later.

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Negative numbers were first explicitly mentioned in the French mathematician Nicolas Chuquet's (1445–1500) work Le Triparty en la science des nombres (1484; published in Lyon in 1848). However, initial notions of negative numbers already existed in the works of Indian and Chinese mathematicians. For example, Chinese mathematicians used negative numbers implicitly when solving systems of five linear equations with five unknowns.
The Indian mathematician Brahmagupta (598–660) described negative numbers as "debts." He stated the following rules: "The sum of two debts is a debt." "The sum of zero and a debt is a debt." He referred to a positive number as a "property," thus defining the sum of "property" and "debt" as their difference. If they are equal, the result is zero.
Arab mathematicians used metaphors: negative signs as "enemies" and positive signs as "friends," and they interpreted the signs of the product of numbers with real-life rules. In the field of irrational numbers, Persian mathematician al-Karaji (died 1016) in his book Al-Fakhri discussed evaluating polynomials containing square and cube roots. He also performed transformations on simple cube roots, such as simplifying expressions like $\sqrt{a} + \sqrt{b}$.
The term "rational" comes from the Latin ratio, meaning "ratio," and "irrational" means not rational. Originally, these terms were applied to measurable and immeasurable quantities. Roman mathematicians Martianus Capella and Cassiodorus in the 5th and 6th centuries translated these terms into Latin as rational and irrational, respectively.
In Euclid's Elements, irrational numbers are discussed from a geometric perspective. By the beginning of the Common Era, unlike Greek geometric algebra, in the Eastern countries both geometry and arithmetic-based algebra began to develop rapidly. Plane and spherical trigonometry and the computational methods needed for astronomy were also improved.
Despite the fact that Eastern mathematicians in India, Central Asia, and the Near East could not work without irrational numbers while developing algebra, trigonometry, and astronomy, they still hesitated to fully accept these numbers. The Greeks called irrational quantities alogos (unspeakable), and the Arabs referred to them as asami(mute).

In the 16th century, Italian mathematician Rafael Bombelli (1526–1572) and Dutch mathematician Simon Stevin (1548–1620) considered irrational numbers to be even more powerful than rational ones.
Even before them, many mathematicians of the Near and Far East had widely used irrational numbers in algebra. For example, Omar Khayyam, in his work Commentaries on Difficult Postulates of Euclid, introduced the idea of divisible units and a generalized number concept, referring to them as "numbers." This generalized concept included both rational and irrational numbers.
Thus, Omar Khayyam modernized the ancient concept of numbers, defining ratios of quantities as numbers themselves. These ratios were the new kind of numbers—rational in the old sense but general numbers in the new sense.

Overall, Khayyam showed that there is no essential difference between irrational quantities and numbers, thereby expanding the concept of numbers to positive real numbers.

In this field, the Azerbaijani mathematician Nasir al-Din al-Tusi (1201–1274) also made significant contributions. In his works Treatise on the Complete Quadrilateral and Commentary on Euclid, he further developed the theory of proportions and teachings about numbers.

His book Commentary on Euclid (Tahrir Uqlidis), which was renowned in both the East and later in medieval Europe, exists in two versions: one brief and another extended version with 10 books, published in Rome in 1594. In it, the scholar elaborates on square irrationalities and gives the following definition of a rational quantity: "Any quantity that is in ratio with a given quantity is called rational, wherein a number is in ratio with another number." Otherwise, it is called irrational. An irrational quantity, in relation to another quantity, is like the ratio of a number to another when the first is irrational. For example:

$\sqrt{2}$ or $\sqrt{3}$

In Europe, Simon Stevin wrote about decimal fractions about 150 years after Al-Kashi, in 1585. In 1594, in another work Algebraic Supplements, he developed the ideas from his earlier work and showed that decimal fractions could be used to approximate real numbers infinitely closely. Thus, in the 16th century, the introduction and formal justification of the concept of irrational numbers led to the creation of the idea of decimal computation.

The publication of the book Geometry (1637) by the great French philosopher, mathematician, physicist, and physiologist René Descartes (1596–1650) helped clarify the link between irrational numbers and the measurement of arbitrary segments. On the number line, irrational numbers were represented as points, just like rational numbers. This geometric representation made it easier to understand the nature of irrational numbers and facilitated their acceptance.

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