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NONLINEAR OSCILLATIONS OF A BODY MOUNTED ON VISCOELASTIC OSCILLATING SUPPORTS

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Abstract: This paper examines the nonlinear oscillations of a rigid body mounted on viscoelastic supports. The equations of motion of the system are derived from Lagrange's equations of the second kind for systems with a finite number of degrees of freedom. A solution method for the problem is developed, numerical results are obtained, and the influence of nonlinearity on displacement amplitudes is evaluated.

Keywords: nonlinear oscillations, rigid body, vibration, Lagrange equations, viscoelastic support, degree of freedom

1. Introduction

Issues related to dynamic vibration damping of mechanical systems are widely applied in machine dynamics [1,2]. The use of passive vibration reduction methods is especially effective for objects exposed to steady periodic external forces. Developments are known that are associated with the creation of complex vibration reduction systems under multiple disturbances [3,4].

Theoretical approaches to solving vibration reduction problems based on structural methods of mathematical modeling [5,6] are of certain interest. Within this framework, a mechanical oscillatory system is interpreted as a certain dynamic automatic control system. In this case, the structural diagram of the system acts as a structural analogue of the initial mathematical model, obtained in the form of a system of differential equations [7,8].

In the structural scheme of a vibration-protection system, the object subject to vibration effects can be highlighted through structural transformations as a second-order integrating element when the structure of the mechanical oscillatory system (or vibration-protection system) is reduced to a scheme consisting of the protected object and its surrounding negative feedback loop. This makes it possible to determine dynamic damping regimes based on the properties of the transfer functions of the feedback circuits [9,10].

In the physical sense, the negative feedback relative to the protected object in the structural model generally reflects the elastic properties of the vibration-protection system or the equivalent stiffness, depending on the frequency of external excitation.

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2. Methods

2.1. Problem statement and solution methods

Let us consider the oscillations of a rigid body mounted on viscoelastic supports. The mechanical system shown in Fig. 1 consists of a rigid body (1) mounted on a rigid plate (2) by means of viscoelastic supports (springs) (3). We determine the amplitude–frequency characteristics of various points of the body under a given harmonic law of oscillations of the supporting plate.

The equations of motion of the system are derived from Lagrange's equations of the second kind for a system with elastic supports, replacing the stiffnesses of the supports with integral operators [6].

$$m\ddot{x}_c + c_x x_c - cl\varphi - \varepsilon_1 \; c_x \int_{-\infty}^{t} R_x(t-\tau)x_c(\tau)d\tau + c_x l \int_{-\infty}^{t} R_x(t-\tau)\varphi(\tau)d\tau \; =$$

$$= A\sin\nu t + \varepsilon \frac{c_x l}{6} \; \varphi^3 - \int_{-\infty}^{t} R_x(t-\tau)\varphi^3(\tau)d\tau \; ,$$

$$I\ddot{\varphi} - c_x l x_c - \left(c_\varphi + c_x l^2\right)\varphi + \varepsilon_1 \; c_x l \int_{-\infty}^{t} R_x(t-\tau)x_c(\tau)d\tau - \int_{-\infty}^{t} c_\varphi R_\varphi(t-\tau) + c_x l^2 R_\varphi(t-\tau) \; \varphi(\tau)d\tau \; = \quad (1)$$

$$= \varepsilon \; -\frac{c_x l}{2} \; \varphi^2 x_c - \int_{-\infty}^{t} R_x(t-\tau)\varphi^2(\tau)x_c(\tau)d\tau \; + \frac{8}{6} \; \varphi^3 - \int_{-\infty}^{t} R_x(t-\tau)\varphi^3(\tau)d\tau \; .$$

where c_x, c_φ — the stiffnesses of the supports in the direction of two generalized coordinates; I — the moment of inertia, R_x и R_φ — relaxation kernels in the x and φ directions; m — mass of the body.

The generating system (1) at $\varepsilon = 0, \;\; \varepsilon_1 = 0$ has frequencies

$$\omega_{1,2}^2 = \frac{1}{2} \; \frac{c_x l^2 + c_\varphi}{I} + \frac{c_x}{m} \; \pm \sqrt{\frac{1}{4} \; \frac{c_x l^2 + c_\varphi}{I} + \frac{c_x}{m} \; ^2 - \frac{c_x c_\varphi}{mI}}. \quad (2)$$

Let us perform, for (1), a transformation to normal coordinates, representing the solution as an expansion in normal modes:

$$x = a_{11} f_1 + a_{12} f_2, \quad \varphi = a_{21} f_1 + a_{22} f_2. \quad (3)$$

Let us satisfy the equations of oscillations (1) at $\varepsilon = 0, \;\; \varepsilon_1 = 0$ with functions

$$\begin{aligned} &x = a_{11}\sin\left(\omega_1 t + \alpha_1\right), \quad x = a_{21}\sin\left(\omega_2 t + \alpha_2\right), \\ &\varphi = a_{21}\sin\left(\omega_1 t + \alpha_1\right), \quad \varphi = a_{22}\sin\left(\omega_2 t + \alpha_2\right), \end{aligned} \quad (4)$$

where $a_{11}, a_{12}, a_{21}, a_{22}$ – the desired amplitudes.

Substituting (4) into (1), we obtain the auxiliary

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relations

$$
\begin{aligned}
&-ma_{11}\omega_1^2 + c_x l a_{11} - c_x l a_{21} = 0,\\
&-Ia_{21}\omega_1^2 - c_x l a_{11} + \left(c_x l^2 + c_c\right)a_{21} = 0,\\
&-ma_{12}\omega_2^2 + c_x l a_{12} - c_x l a_{22} = 0,\\
&-Ia_{22}\omega_2^2 - c_x l a_{12} + \left(c_x l^2 + c_c\right)a_{22} = 0,
\end{aligned} \qquad (5)
$$

which make it possible to determine the amplitudes $a_{11}, a_{12}, a_{21}, a_{22}$ – in the form

$$
\begin{aligned}
&a_{21} = \frac{m\omega_1^2 + c_x x_c + c_x l}{c_x l^2 + c_x l + c_\varphi - I\,\omega_1^2},\\
&a_{22} = -\frac{m\omega_2^2 + c_x l - c_x l}{c_x l^2 - c_x + c_\varphi - I\,\omega_2^2}, \qquad (6)\\
&a_{11} = a_{12} = 1.
\end{aligned}
$$

Thus, system (1), taking into account (3) and (5), takes the form

$$\ddot{f}_1 + \Omega_2 \;\; f_1 - \int_{-}^{t} R_1\left(t-\tau\right) f_1\left(\tau\right) d\tau \;\; = P\sin\nu t + F_1\left(f_1, f_2\right), \qquad (7)$$

$$\ddot{f}_2 + \Omega^2 \;\; f_2 - \int_{-}^{t} R_2\left(t-\tau\right) f_2\left(\tau\right) d\tau \;\; = F_2\left(f_1, f_2\right), \qquad (8)$$

The periodic solutions (7) and (8), according to [7, 8], can be expressed as follows:

$$f_1 = f_{1,0} + \sum_{n=1} \;\; f_{1,n} - f_{1,n(n-1)} \;\; , \qquad (9)$$

$$f_2 = f_{2,0} + \sum_{n=1} \;\; f_{2,n} - f_{2,n(n-1)} \;\; , \qquad (10)$$

By substituting (9) and (10) respectively into equations (7) and (8), we obtain the system of equations [9]

$$
\begin{aligned}
&\ddot{f}_{1,n} + \Omega^2 \;\; f_{1,n} - \int_{-}^{t} R_1\left(t-\tau\right) f_{1,n}\left(\tau\right) d\tau \;\; = P\sin\nu t + F_1\left(f_{1,(n-1)}, f_{2,(n-1)}\right),\\
&\ddot{f}_{2,n} + \Omega^2 \;\; f_{2,n} - \int_{-}^{t} R_2\left(t-\tau\right) f_{2,n}\left(\tau\right) d\tau \;\; = F_2\left(f_{1,(n-1)}, f_{2,(n-1)}\right).
\end{aligned} \qquad (11)
$$

Пусть функции $F_1\left(f_1, f_2\right)$ и $F_2\left(f_1, f_2\right)$ представимы в виде равномерно сходящихся рядов Фурье:

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$$
\begin{aligned}
F_1(f_1, f_2) &= \sum_{k_1=0} \psi_{k_1} \sin(\mu_k t + \nu_k), \\
F_2(f_1, f_2) &= \sum_{k_2=0} \psi_{k_2} \sin(\mu_k t + \nu_k),
\end{aligned}
\qquad (12)
$$

Solution of equations (7), (8) under the conditions $F_1(f_1, f_2) = 0$ and $F_2(f_1, f_2) = 0$ will:

$$
\begin{aligned}
f_1(t) &= \sum_{k_1=0} c_{k_1} \sin(\mu_k t + \beta_k), \\
f_2(t) &= \sum_{k_2=0} c_{k_2} \sin(\mu_k t + \beta_k),
\end{aligned}
\qquad (13)
$$

Where

$$
\begin{aligned}
c_{k_1} &= \frac{F_{k_1}}{\left[\Omega^4 R_{sk} + \left(\Omega^2\left(1 - R'_{ck}\right) - \mu_k^2\right)^2\right]^{1/2}}, \\
c_{k_2} &= \frac{F_{k_2}}{\left[\Omega^4 R_{sk} + \left(\Omega^2\left(1 - R'_{ck}\right) - \mu_k^2\right)^2\right]^{1/2}},
\end{aligned}
\qquad (14)
$$

$$
R'_{sk} = \int_0 R(\tau)\sin\mu_k\tau d\tau; \qquad R'_{ck} = \int_0 R(\tau)\cos\mu_k\tau d\tau; \qquad (*)
$$

Where R'_{sk} and R'_{ck} are defined by expressions (*); the constants β_k satisfy the equation

$$
tg(\gamma_k - \beta_k) = \frac{\Omega^2 R_{sk}}{\Omega^2(1 - R_{ck}) - \mu_k^2}. \qquad (15)
$$

The series (13) converges absolutely and uniformly provided that $0 < R_c < 1$. Function (13) does not contain arbitrary constants and characterizes forced oscillations in the presence of periodic disturbances (12).

Let us consider nonlinear integro-differential equations (7-8), representing them in the following form:

$$
\begin{aligned}
\ddot{f}_{1n} + \Omega_1^2 \left[f_{1n} - \int_{-}^{t} R_1(t-\tau) f_{1n}(\tau) d\tau \right] &= P_1 \sin\nu t + \mu F_{1n}(f_1, f_2), \\
\ddot{f}_{2n} + \Omega_2^2 \left[f_{2n} - \int_{-}^{t} R_2(t-\tau) f_{2n}(\tau) d\tau \right] &= P_2 \sin\nu t + \mu F_{2n}(f_1, f_2);
\end{aligned}
\qquad (16)
$$

Here f_{1n} and f_{2n} - desired functions; known constant; $F_{1n}(f_1, f_2)$ and $F_{2n}(f_1, f_2)$ – known functions; μ – dimensionless positive parameter.

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We assume that the kernel R satisfies the condition

$$0 < \int_{0}^{\infty} R(\tau) d\tau < 1, \qquad (17)$$

and functions $F_{1n}(f_1, f_2)$ and $F_{2n}(f_1, f_2)$ can be represented as an infinite series (12).

Consequently, the linear equations (19) are the result of applying the method of successive approximations [10] to the original nonlinear equation (16). Since the right-hand sides of equation (19) are periodic functions of time, we find its solution in the form (13), and the right-hand sides of (19) should be expanded in a Fourier series.

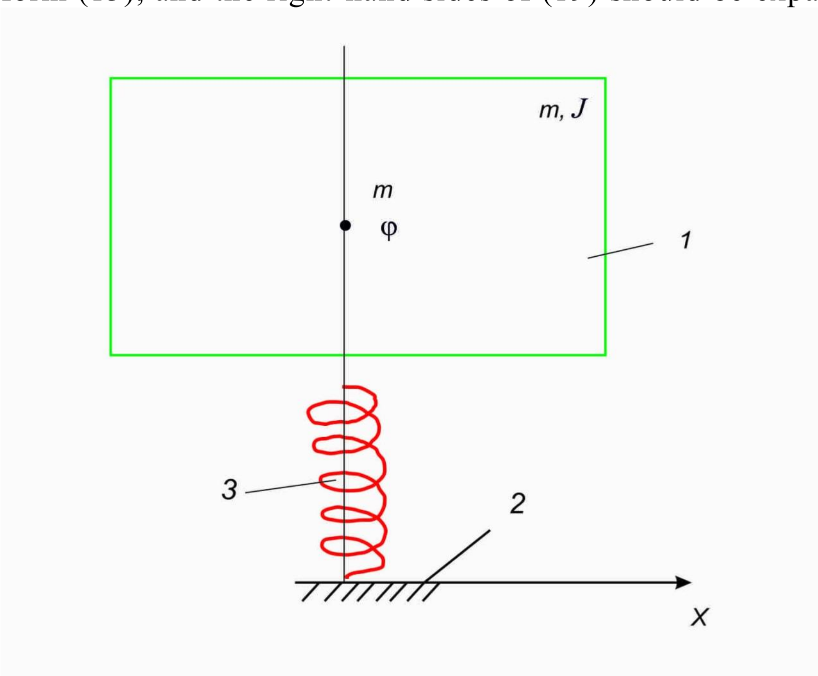

Fig. 1. Calculation scheme: 1-solid body, 2-base, 3-spring

Equations (19) can be transformed in the following sequence

$$\ddot{f}_1 - \ddot{f}_{01} + \Omega^2 \left(f_1 - f_{01} - \int_{-\infty}^{t} R(t-\tau)(f_1 - f_{01}) d\tau \right) = \mu F(f_{01}, f_{02}, t),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (20)$$

$$\ddot{f}_k - \ddot{f}_{k-1} + \Omega^2 \left(f_k - f_{k-1} - \int_{-\infty}^{t} R(t-\tau)(f_k - f_{k-1}) d\tau \right) = \mu F(f_{1,k-1}, f_{2,k-1}, t).$$

3. Results and analysis

Estimates of periodic solutions of equations (20) are given in [11].

When solving specific problems, the following mechanical characteristics of the body were adopted: m=1; l=0.5; I=0.1. The relaxation core is taken as

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$$R(t-s)=\frac{Ae^{-\beta(t-s)}}{(t-s)^{1-\alpha}}.$$

Two variants of kernel parameters are considered (21):

1) $\alpha=0,1;$ $A=0.078;$ $\beta=0.05$

2) $\alpha=0,1;$ $A=0.048;$ $\beta=0.05$

The results found for low and high viscosities are qualitatively the same, differing only in the significantly larger values of the amplitudes for low viscosity. Therefore, the analysis of the solutions obtained is given only for high viscosity..

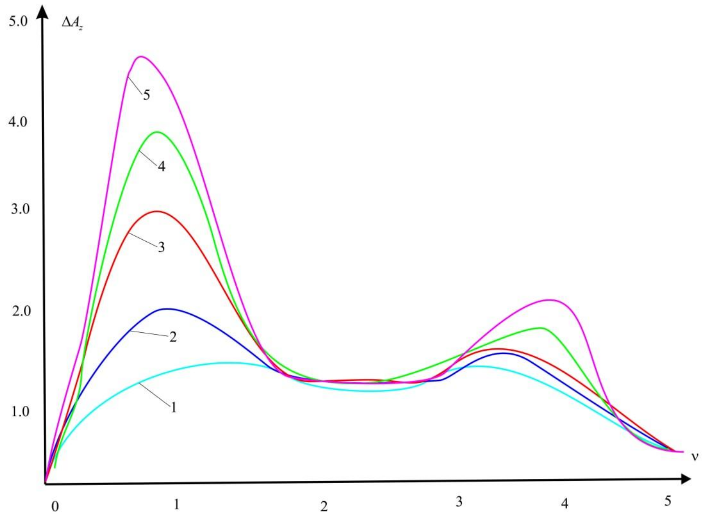

Fig. 2. Change in the amplitude of displacements from the frequency of external loads:1.A=0.078; 2. A=0.048; 3.A=0.030; 4.A=0.020;A=0.015

Fig. 2 shows the calculation results for nonlinear problems. According to the numerical results, the amplitude value for a nonlinear problem is 3-4% greater than for a linear problem..

4. Conclusions

The paper presents methods for solving the problem of nonlinear oscillations of a mechanical system with two degrees of freedom. Based on the numerical results obtained, it was found that the amplitude value for a nonlinear problem is 3-4% greater than for a linear problem.

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Literature:

1. Vibrations in technology: Handbook: In 6 volumes. Vol. 6. Protection from vibrations and shocks M.: Mechanical Engineering, 1981. 456 p.

2. Tokarev M. F., Talitsky E. N., Frolov V. A. Mechanical effects and protection of electronic equipment: Textbook for universities. Moscow: Radio and Communications, 1984. 224 p.

3. Nashif A., Jones D., Henderson J. Vibration damping. Moscow: Mir Publ., 1988.-448 p.

4. Teshaev M. K., Safarov I. I., Mirsaidov M. Oscillations of multilayer viscoelastic composite toroidal pipes // Journal of the Serbian Society for Computational Mechanics. 2019. Vol. 13, No. 2. P. 104-115. DOI: 10.24874/jsscm.2019.13.02.08.

5. Gludkin O. P. Methods and devices of testing RES and EVS. M.: Higher School, 1991.-336s.

6. Gludkin O. P., Engalychev A. N., Korobov A. I., Tregubov Yu. V. Tests of radioelectronic, electronic computing equipment and test equipment. Moscow: Radio and Communications, 1987. 272 p.

7. Lysenko A.V., Goryachev N. V., Grab I. D., Kemalov B. K., Yurkov N. K. A brief overview of simulation modeling methods // Modern information technologies. 2011. No. 14. pp. 171-176.