JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

GEOMETRIC MODELING OF VIRTUAL MUSEUM EXHIBITS USING THE PIECEWISE POLYNOMIAL METHOD

Khudayberganov Temur Rustamovich
Associate professor in Asia international University

Abstract: In the realm of digital preservation and virtual representation of cultural artifacts, the application of geometric modeling techniques holds significant promise. This paper focuses on the utilization of the piecewise polynomial method for the accurate and detailed modeling of museum exhibits within virtual environments. By leveraging the piecewise polynomial approach, this research aims to provide a comprehensive analysis of how intricate shapes and structures of various museum artifacts can be faithfully reconstructed and interactively presented in the digital space. Furthermore, the study delves into the technical aspects of implementing and optimizing this method for virtual museum applications. Through this rigorous exploration, the research seeks to not only demonstrate the potential of the piecewise polynomial method but also contribute to the advancement of virtual museum experiences, digital conservation, and educational outreach initiatives.

Keywords: Geometric modeling, virtual museum, piecewise polynomial method, cultural heritage preservation, digital representation, interactive exhibits, 3d modeling, digital conservation, cultural artifacts.

Аннотация: В области цифрового сохранения и виртуального представления культурных артефактов применение методов геометрического моделирования имеет значительные перспективы. В данной статье основное внимание уделяется использованию кусочно-полиномиального метода для точного и детального моделирования музейных экспонатов в виртуальной среде. Используя кусочно-полиномиальный подход, это исследование направлено на всесторонний анализ того, как сложные формы и структуры различных музейных экспонатов могут быть достоверно реконструированы и интерактивно представлены в цифровом пространстве. Кроме того, в исследовании рассматриваются технические аспекты внедрения и оптимизации этого метода в приложениях для виртуальных музеев. В ходе этого тщательного исследования автор стремится не только продемонстрировать потенциал метода кусочно-полиномиального анализа, но и внести свой вклад в развитие опыта работы с виртуальными музеями и просветительские инициативы.

Ключевые слова: Геометрическое моделирование, виртуальный музей, кусочно-полиномиальный метод, сохранение культурного наследия, цифровое представление, интерактивные экспонаты, 3d-моделирование, культурные артефакты.

Annotatsiya: Madaniy yodgorliklarni raqamli ko‘rinishda saqlash va virtual taqdim etish sohasida geometrik modellashtirish usullarini qo'llash katta istiqbolga ega. Ushbu maqola virtual muhitda muzey eksponatlarini aniq va batafsil modellashtirish uchun bo‘lakli ko‘phad usulidan foydalanishga qaratilgan. Bo‘lakli ko‘phad yondashuvidan foydalangan holda, ushbu tadqiqot turli muzey eksponatlarining murakkab shakllari va tuzilmalarini raqamli makonda qanday qilib ishonchli tarzda qayta qurish va interaktiv tarzda taqdim etish mumkinligini har tomonlama tahlil qilishga qaratilgan. Bundan tashqari, tadqiqot virtual muzey ilovalarida ushbu usulni amalga oshirish va optimallashtirishning texnik jihatlarini ko'rib chiqadi. Ushbu tadqiqot davomida muallif nafaqat bo‘lakli ko‘phad tahlil qilish usulining imkoniyatlarini namoyish etishga, balki virtual muzeylar va ta'lim tashabbuslari bilan tajriba orttirishga hissa qo'shishga intiladi.

Kalitli so’zlar: *Geometrik modellashtirish, virtual muzey, qismli polinom usuli, madaniy merosni saqlash, raqamli vakillik, interaktiv eksponatlar, 3d modellashtirish, madaniy asarlar.*

I. INTRODUCTION

After our republic gained independence, there has been an introduction of virtual reality environments by both state and private organizations to study culture, art, and historical monuments. The content of exhibits is displayed in three-dimensional and video electronic formats, and a virtual museum is designed using engineering geometry and computer graphics. The development of a national virtual environment for museums is receiving special attention. The President of the Republic of Uzbekistan has approved the "Digital Uzbekistan-2030" strategy and effective measures for its implementation. This includes the use of virtual reality, artificial intelligence, machine learning, and cloud computing in various economic sectors. The strategy also aims to promote scientific research in digital technologies and improve organizational mechanisms. As technology continues to advance, a lot of people are becoming increasingly interested in exploring the virtual world of museums. Also, the demand for an electronic catalog to save a copy of cultural and artistic, spiritual and historical monuments based on applied geometry and computer graphics based on virtual reality tools has increased, and it is important to develop software tools that automate virtual environments with these types of methods, digital models and algorithms. is one of the tasks.[1]

The exploration and presentation of museum exhibits in the virtual domain have become a significant area of interest in the digital age, blending the realms of cultural heritage with advanced computational techniques. One such cutting-edge approach to creating realistic and interactive representations of artifacts and historical pieces is through geometric modeling, specifically using the piecewise polynomial method. This technique allows for the meticulous recreation of objects in a virtual environment, offering a detailed and immersive experience for users worldwide.

Geometric modeling plays a crucial role in the digital preservation and dissemination of cultural heritage, enabling not only the safeguarding of delicate and priceless artifacts but also providing an accessible platform for education and research. The piecewise polynomial method, in particular, offers a robust framework for modeling complex shapes and surfaces with high precision. By segmenting the object into smaller, manageable pieces and approximating each segment with polynomial functions, this method can capture the intricate details and textures of museum exhibits with remarkable accuracy.

The introduction of the piecewise polynomial method into the realm of virtual museum exhibits opens up new avenues for exploration and interaction. Users can virtually navigate through collections, examine artifacts from multiple angles, and even experience historical contexts

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

through interactive scenarios. This not only enriches the user experience but also extends the reach of museum collections, making them available to a global audience without the constraints of physical location or accessibility.

Moreover, the application of geometric modeling using the piecewise polynomial method in virtual museum exhibits holds significant potential for research and education. It enables scholars to analyze and study artifacts in detail without the risk of physical handling, preserving the artifacts' integrity while still allowing for in-depth examination. Additionally, it provides an innovative tool for educators to engage students with history and culture in a more interactive and engaging manner.

In summary, the geometric modeling of virtual museum exhibits using the piecewise polynomial method represents a convergence of technology and cultural heritage, offering new possibilities for preservation, education, and accessibility. This approach not only enhances the way we interact with museum collections but also plays a vital role in the ongoing effort to safeguard and share our global cultural heritage in the digital era.

By the general public, museologist, archeologist and historian scientists use high-resolution video, photorealistic electronic resources in the virtual world, the place, value, and adequacy level of categorization of materials in virtual electronic format in the process of museum exhibits based on engineering geometry and computer graphics, adding elements of virtual reality in the collection system. Yu.B. Bloxinova, I.G. Jurkina, V.A. Knyazya, A.N. Lobanova, A.P. Mikhaylova, A.G. Chibunicheva, Th. Luhmann, B. Keith Atkinson, Armin Gruen from foreign scientists on development of application systems, virtual environments and their implementation. , Richard Hartley, Matt Weilberg, and others' scientific work is noteworthy.

Application of virtual reality elements based on engineering geometry and computer graphics in the museum system of our republic, application of methods and algorithms for visualization of three-dimensional models of objects for the virtual environment, scientific research on software information system of virtual museum process automation, mainly A.Kh. Nishanov, J.Kh. Djumanov, F.M. Nuraliyev, A.Sh. Mukhammadiyev, Sh.A.Anarova, R.D.Aloyev, U.R.Khamdamov and others have been making their contributions. Currently, one of the urgent problems in the virtual world is that the digital collection fund of the museum in the virtual environment, the creation of 3D models of them and the development of the software tool, the integrated museum with automated information systems, the technologies of online visit to the virtual museum environment through the Internet are not sufficiently studied.

II. METHODS

We express the analytical view of the geometric model in the form of a third-order polynomial of the $Y(x) = y_i(x)$ function in each section [xi-1, xi] (i=1,2,…,N):

$$y_i(x)=a_i+b_i(x-x_i)+c_i(x-x_i)^2++d_i(x-x_i)^3 \qquad (1)$$

$x_{i-1} \leq x \leq x_i$, (i=0,1,2,…,N)

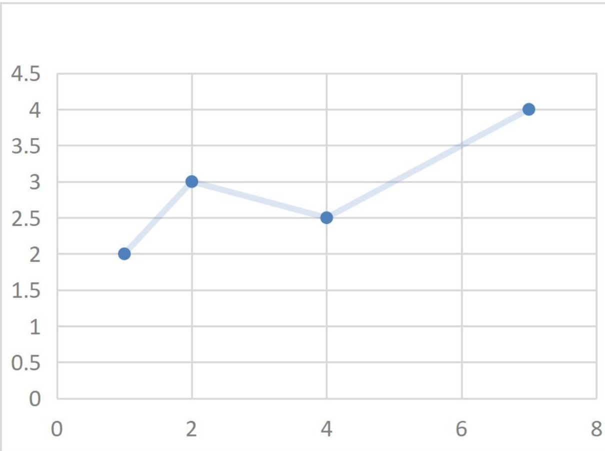
JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

where a, b, c, d are coefficients, these coefficients are found by taking derivatives from formula (1):

Given the following points (table 1):

Table 1. Given points.

№	x	y
0	1	2
1	2	3
2	4	2.5
3	7	4



Picture 1. Given points

We interpolate through these points using formula (1). We write down the formulas for the three-part polynomial

$$\begin{cases} y_0 = a_0 + b_0(x-x_0) + c_0(x-x_0)^2 + d_0(x-x_0)^3 \\ y_1 = a_1 + b_1(x-x_1) + c_1(x-x_1)^2 + d_1(x-x_1)^3 \\ y_2 = a_2 + b_2(x-x_2) + c_2(x-x_2)^2 + d_2(x-x_2)^3 \end{cases}$$

After entering the appropriate coordinates, it will look like this:

$$\begin{cases} y_0 = a_0 + b_0(x-1) + c_0(x-1)^2 + d_0(x-1)^3 \\ y_1 = a_1 + b_1(x-2) + c_1(x-2)^2 + d_1(x-2)^3 \\ y_2 = a_2 + b_2(x-4) + c_2(x-4)^2 + d_2(x-4)^3 \end{cases} \quad (2)$$

We determine the equation by substituting the values of x and y through the points below.

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

For the 1st polynomial $x_0=1$, $y_0=2$, $x_1=2$, $y_1=3$:

$y_0=a_0+b_0(x-1)+c_0(x-1)^2+d_0(x-1)^3$,

$$\begin{cases} 2 = a_0+b_0(1-1)+c_0(1-1)^2+d_0(1-1)^3 \\ 3 = a_0+b_0(2-1)+c_0(2-1)^2+d_0(2-1)^3 \end{cases}$$

$$\begin{cases} a_0=2 \\ a_0+b_0+c_0+d_0=3 \end{cases}$$

For the 2nd polynomial $x_1=2$, $y_1=3$, $x_2=4$, $y_2=2.5$:

$y_1=a_1+b_1(x-2)+c_1(x-2)^2+d_1(x-2)^3$,

$$\begin{cases} 3=a_1+b_1(2-2)+c_1(2-2)^2+d_1(2-2)^3 \\ 2{,}5=a_1+b_1(4-2)+c_1(4-2)^2+d_1(4-2)^3 \end{cases}$$

$$\begin{cases} a_1=3 \\ a_1+2b_1+4c_1+8d_1=2{,}5 \end{cases}$$

For the 3th polynomial $x_2=4$, $y_2=2.5$, $x_3=7$, $y_3=4$:

$y_2=a_2+b_2(x-4)+c_2(x-4)^2+d_2(x-4)^3$,

$$\begin{cases} 2{,}5=a_2+b_2(4-4)+c_2(4-4)^2+d_2(4-4)^3 \\ 4=a_2+b_2(7-4)+c_2(7-4)^2+d_2(7-4)^3 \end{cases}$$

$$\begin{cases} a_2 = 2{,}5 \\ a_2+3b_2+9c_2+27d_2=4 \end{cases}$$

In order for a polynomial line passing through each point to be smooth, the attempts to enter and exit the curve must overlap. for this

$y_0'= y_1',\ y_0''= y_1''\ ,\ y_1'= y_2'\ ,\ y_1''= y_2''$ (3)

equals must be met. We find the 1st and 2nd derivatives of every polynomial.

$$\begin{cases} y_0'=(a_0+b_0(x-1)+c_0(x-1)^2+d_0(x-1)^3)'= \\ \quad =b_0+2c_0(x-1)+3d_0(x-1)^2 \\ y_1'=(a_1+b_1(x-2)+c_1(x-2)^2+d_1(x-2)^3)'= \\ \quad =b_1+2c_1(x-2)+3d_1(x-2)^2 \\ y_2'= (a_2+b_2(x-4)+c_2(x-4)^2+d_2(x-4)^3)'= \\ \quad = b_2+2c_2(x-4)+3d_2(x-4)^2 \end{cases}$$

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

$$\begin{cases} y_0''=(b_0+2c_0(x-1)+3d_0(x-1)^2)'= \\ \quad = 2c_0+6d_0(x-1) \\ y_1''=(b_1+2c_1(x-2)+3d_1(x-2)^2)'= \\ \quad = 2c_1+6d_1(x-2) \\ y_2''= (b_2+2c_2(x-4)+3d_2(x-4)^2)'= \\ \quad = 2c_2+6d_2(x-4) \end{cases}$$

$y_0'= y_1', y_0''= y_1''$ **for case** $x_1=2$, $y_1=3$:

$$\begin{cases} b_0+2c_0(x-1)+3d_0(x-1)^2= \\ = b_1+2c_1(x-2)+3d_1(x-2)^2 \\ 2c_0+6d_0(x-1) = 2c_1+6d_1(x-2) \end{cases}$$

$$\begin{cases} b_0+2c_0+3d_0- b_1=0 \\ 2c_0+6d_0- 2c_1 = 0 \end{cases}$$

$y_1'= y_2'$, $y_1''= y_2''$ **for case** $x_2=4$, $y_2=2.5$:

$$\begin{cases} b_1+2c_1(x-2)+3d_1(x-2)^2= \\ =b_2+2c_2(x-4)+3d_2(x-4)^2 \\ 2c_1+6d_1(x-2)= 2c_2+6d_2(x-4) \end{cases}$$

$$\begin{cases} b_1+4c_1+12d_1- b_2=0 \\ 2c_1+12d_1- 2c_2=0 \end{cases}$$

We determine the property of polynomials at the initial and final points. We give zero curvature to polynomials.

$y_0''=0$, $y_2''=0$, $x_0=1$, $y_0=2$, $x_3=7$, $y_3=4$ **(4)**

$$\begin{cases} 2c_0+6d_0(x-1)=0 \\ 2c_2+6d_2(x-4)=0 \end{cases}$$

$$\begin{cases} 2c_0=0 \\ 2c_2+18d_2=0 \end{cases}$$

We will have a system of 12 linear equations and simplify to 8

$$\begin{cases} a_0=2 \\ a_1=3 \\ a_2=2{,}5 \\ c_0=0 \\ b_0+d_0=1 \\ 2b_1+4c_1+8d_1=-0.5 \\ 3b_2+9c_2+27d_2=3 \\ b_0+3d_0-b_1=0 \\ 6d_0-2c_1=0 \\ b_1+4c_1+12d_1-b_2=0 \\ 2c_1+12d_1-2c_2=0 \\ 2c_2+18d_2=0 \\ - \end{cases}$$

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

We find the coefficients by performing the necessary calculations (table 2a)

Table 2. Tables of coefficients

b0	1,428571
d0	-0,42857
b1	0,142857
c1	-1,28571
d1	0,357143
b2	-0,71429
c2	0,857143
d2	-0,09524

a) coefficients

a0	2
b0	1,4285714
c0	0
d0	-0,428571

b) for 1st polynomial

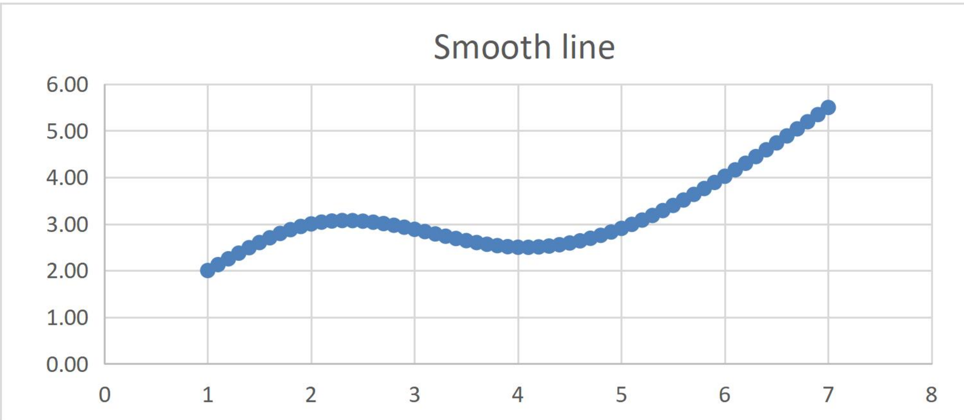
a1	3
b1	0,142857
c1	-1,28571
d1	0,357143

c) for 2nd polynomial

a2	2,5
b2	-0,71429
c2	0,857143
d2	-0,09524

d) for 3rd polynomial

4 coefficients were known in advance. Let's summarize everything (Table 2-b-c-d). We calculate the values of the function with a step of 0.1 by replacing the coefficients in table 4-b-c-d to the functions in formula (2) defined for each segment.



JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

Figure 2. A smooth line generated through the function values with a step of 0.1.

Now we will consider the solutions of various functions passing through the same 4 points.

Table 3. Standard functions passing through the given 4 points

	Linear	y = 0,2738x + 1,9167
	Exponential	y = 2,0178e0,0919x
	Logarithmic	y = 0,8213ln(x) + 2,0485
	Polynomial	y = 0,0944x3 - 1,0778x2 + 3,5722x - 0,5889

We represent the functions in table 3 in a graphic form

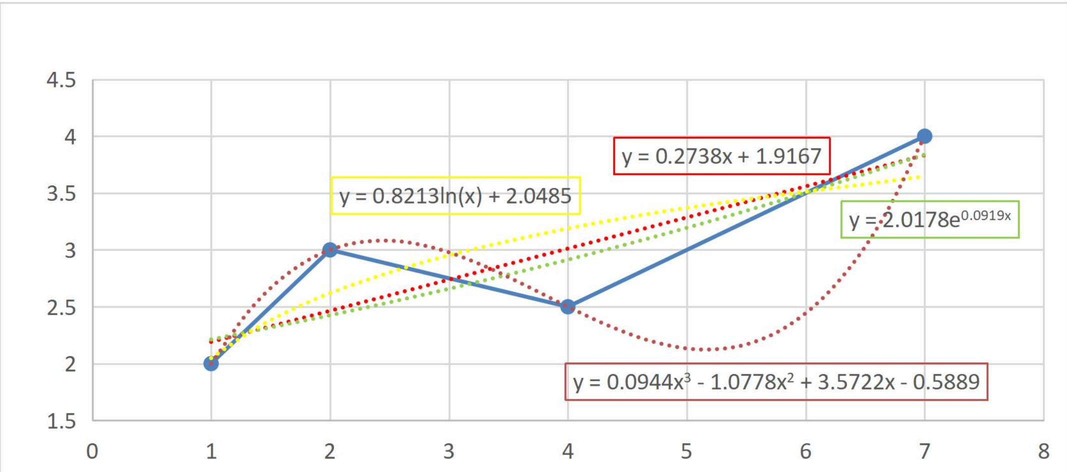


Figure 3. Comparative analysis of curves passing through the first 4 points

It can be seen from the graphs that the curve performed by our proposed piecewise polynomial method is more efficient than the standard functions.

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

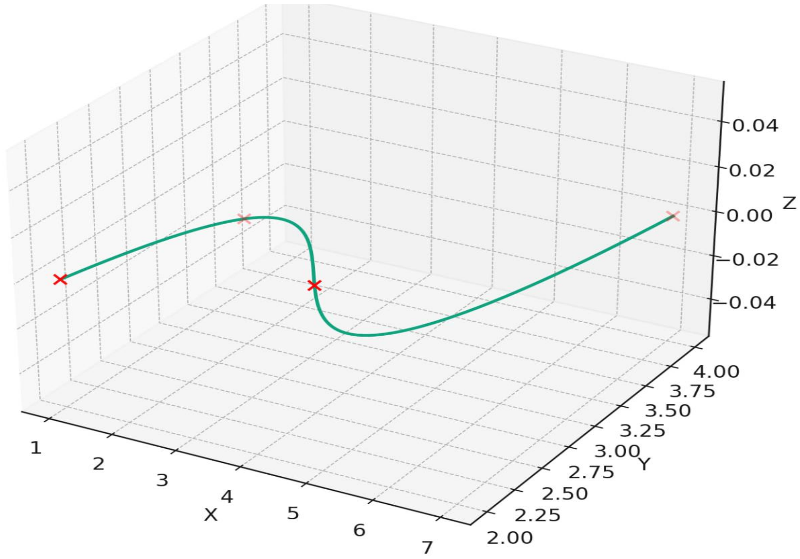
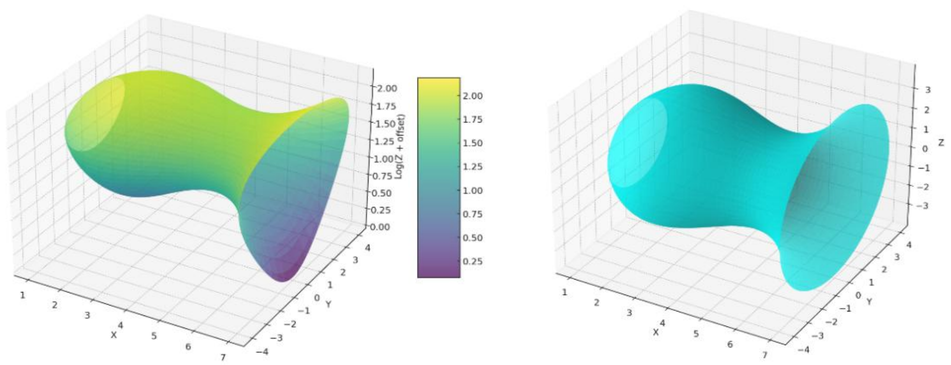


Figure 4. curve representation through given points by the proposed piecewise polynomial method.



a) b)

Figure 5. Through the given points above: a) Case performed by Lograngе spline

b) the model generated by the presented 3rd degree polynomial

After rotating the given points (1,2,0), (2,3,0), (4,2.5,0), (7,4,0) around the X-axis of the object generated using a polynomial To find an analytical solution for the part where y > 0, we first consider the mathematical expression of this object. In this case, we need the exact cubic spline function and its expression after rotation about the X-axis.

Algorithm for intersecting an object with a plane.

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

1. Polynomial function

$Y(x)=a_i+b_i(x-x_i)+c_i(x-x_i)^2+d_i(x-x_i)^3$

2. Rotate around the X axis

After rotating around the x-axis, the new y' and z' coordinates are:

y' = ycos(α)-zsin(α)

z' = ysin(α)+zcos(α)

But since z=0, it looks like below:

y' = ycos(α)

y' = ysin(α)

3. Cutting the object

We use the condition y′>0 for cutting. Based on this condition, we need to consider the part of the cubic spline function y(t) where y>0. For values of y=y(t), y′=y(t)cos(a)>0, which is true for all a for which cos(a)>0.

4. Solution

To write the analytical solution directly, we need an exact representation of the function y(t), but use the given points to calculate the coefficients ai, bi, ci, and di of the polynomial. Based on the given points, it is possible to calculate these coefficients and then analytically express the cut section based on the condition y′>0, but since the calculated values are a large amount of data, its program code is presented and the result is illustrated in fig. 6:

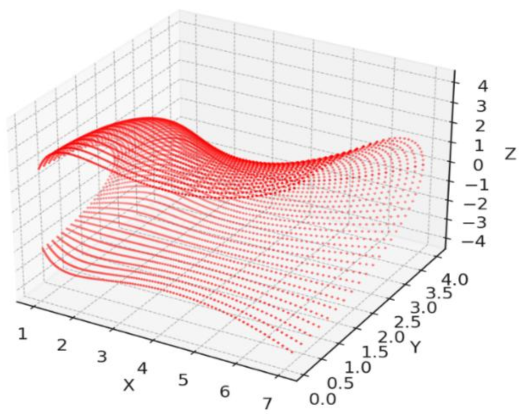


Figure 6. The model created by intersecting the created model with the y = 0 plane

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

It can be seen that, on the basis of the segmented model, we achieve the desired result in contrast to the standard equations. Using formula (1), (3), (4), we draw a curve on the surface of the object (Figure 4). Turn the curve 360 degrees around the x-axis and fill the trajectory with points (Fig. 5).

It can be seen from Figures 4 that the geometric model we propose provides a more accurate solution than the Logrange spline in creating museum exhibits.

III. CONCLUSION

In conclusion, the adoption of geometric modeling using the piecewise polynomial method for virtual museum exhibits marks a significant advancement in the digital preservation and presentation of cultural heritage. This innovative approach has not only transformed the accessibility of museum collections, making them available to a global audience, but has also enhanced the educational and research value of these exhibits. By enabling highly accurate, detailed, and interactive representations of artifacts, the piecewise polynomial method offers a powerful tool for engaging with history and culture in a digital context.

The implications of this technology extend far beyond mere digital replication. It serves as a bridge between the past and the future, allowing us to preserve our cultural treasures in a format that can withstand the test of time and be accessible for generations to come. Moreover, it democratizes access to cultural education, removing physical and geographical barriers that might prevent individuals from experiencing the richness of our global heritage.

Furthermore, the piecewise polynomial method's application in geometric modeling opens up new possibilities for research, enabling scholars to examine artifacts with an unprecedented level of detail and precision. This can lead to new discoveries and insights, contributing to our understanding of history and culture.

As technology continues to evolve, so too will the methods and techniques for digital preservation and presentation. The piecewise polynomial method represents a significant step forward, but it is just the beginning. Continued innovation and development in this field promise even more sophisticated and immersive virtual museum experiences in the future.

Ultimately, the geometric modeling of virtual museum exhibits using the piecewise polynomial method exemplifies the synergy between technology and humanities. It highlights the importance of leveraging digital tools to preserve our cultural heritage, ensuring that it remains vibrant, accessible, and engaging for all.

REFERENCES

[1] Oʻzbekiston Respublikasi Prezidentining 2020-yil 5-oktabrdagi PF-6079-son Farmoni “Raqamli Oʻzbekiston—2030” strategiyasini tasdiqlash va uni samarali amalga oshirish chora-tadbirlari toʻgʻrisida. -T:, https://mitc-dev.egov.uz/news/1778

[2] Djumanov J.X., Xudayberganov T.R., Sobirov B.I., Raximov R.R. “Principles of creation of the virtual museum "dorul hikmat and maorif”. // Electronic journal of actual problems of modern science, education and training. ISSN 2181-9750, Urgench. 2023. № 4. 65-73 pp.

[3] Xudayberganov T.R., Xojiboyev J.M., Boltayev R.Sh. Autodesk 3Ds MAX dasturida

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 15 Issue 10, October 2025
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

modellashtirish. // Высшая школа Научно-практический журнал. Россия. 2018. №14-св3. 36-38 стр.

[4] Голованов Н.Н. Геометрическое моделирование. / М.: Издательсий центр “Академия”, 2011

[5] Роджерс Д. Алгоритмические основы машинной графики: Пер. с англ. – М.: Мир, 1989. – 512 с., ил. ISBN -03-00076-9

[6] Daniel P., Dries N. Report on 3D Publication Formats Suitable for Yeuropeana, 3D Digitisation of Icons of Yeuropean Architectural and Archaeological Heritage. – 2013. – 43 pp.

[7] Yeisert P. Reconstruction of Volumetric 3D Models // 3D Videocommunication: Algorithms, Concepts and Real-Time Systems in Human Centred Communication. – 2006. – P.133 – 150.

[8] Franz-Yerich W., Reuter M., Peinecke N. Geometric Modeling for Yengineering Applications. – 2009. – 53 pp.