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PROPAGATION OF HARMONIC WAVES IN A THIN PANEL (OR PLATE) WITH VARIABLE THICKNESS.

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Abstract: In this work, the vibrations of a thin, long reinforced circular shape under dynamic wave filling, with axial assistance, and considering friction at the contact were investigated using variational theory. They are constructed depending on the period of natural oscillation from wave formation in a circular direction, taking into account friction in the contact between the shell and the fill. It has been established that the research results are practically independent of the material's characteristics, as the frequency dependence on the Poisson's ratio does not depend on the elastic modulus.

Keywords: connection, connection, elastic modulus, deformation, total energy, friction coefficient.

Introduction. A deformed infinite cylindrical shell with thickness is considered h, density ρ, Young's modulus E, Pusson's ratio ν and the viscoelastic properties of the material. In a curvilinear orthogonal coordinate system (α_1 ; α_2 ; z) at $z = 0$ The shell occupies a region.

$$-\quad < \alpha_1 < + \quad ; \quad 0 < \alpha_2 < l; \quad -\frac{h}{2} < z < \frac{h}{2}.$$

Curvature of the middle surface z=0 equal $k_1 = 0; k_2 = \frac{1}{R}$ accordingly. Within the framework of Kirchhoff-Lew hypotheses, the law of change of the components of the displacement vector $u_1^{(z)}$, $u_2^{(z)}$, $w^{(z)}$ shells are determined by the following relations [1,2]

$$u_1^{(z)} = u - \theta_1 z; u_2^{(z)} = v - \theta_2 z \; ; \qquad u_3^{(z)} = w, \qquad (1)$$

where u, v, w – components of the displacement vector of the middle surface; θ_1, θ_2 - angles of rotation of the normal relative to the axes α_1 and α_2.

Forces and moments are related to the deformation components, determined by the relations arising from the generalized Gook's law:

$$T_1 = \tilde{c}(\varepsilon_1 + \nu\varepsilon_2), M_1 = \tilde{D}(x_1 - \nu x_2), \; S = \tilde{A}\varepsilon_{12}; N = \tilde{B}\tau, \qquad (2)$$

where

$$\tilde{c} = \frac{\tilde{E}h}{1-\nu^2}; \quad \tilde{D} = \frac{\tilde{E}h^3}{12(1-\nu^2)}; \quad A = \frac{\tilde{E}h}{2(1+\nu)}; \quad \tilde{B} = \frac{\tilde{E}h^3}{12(1+\nu)};$$

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E – operator modulus of elasticity, which has the form:

$$\tilde{E}\varphi(t) = E_{01}\left[\varphi(t) - \int_{-}^{t} R_E(t-\tau)\varphi(t)\,d\tau\right],$$

$\varphi(t)$ - arbitrary time function; $R_E(t-\tau)$ - relaxation nucleus; E_{01} - instantaneous elasticity modulus; ν - Pusson's ratio.

Methods. After substituting the expression (2) into the equation, the principle of possible displacements and the standard integration procedure by parts, we obtain the equations of motion in the form:

$$\frac{\partial T_1}{\partial \alpha_1} + \frac{\partial S}{\partial \alpha_2} = -\rho h \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial T_2}{\partial \alpha_2} + \frac{\partial S}{\partial \alpha_1} + k_2 Q_2 = -\rho h \frac{\partial^2 \vartheta}{\partial t^2}, \quad \frac{\partial Q_1}{\partial \alpha_1} + \frac{\partial Q_2}{\partial \alpha_2} - k_2 T_2 = -\rho h \frac{\partial^2 w}{\partial t^2}$$

$$Q_1 = \frac{\partial M_1}{\partial \alpha_1}, \quad Q_2 = \frac{\partial M_2}{\partial \alpha_2} + 2\frac{\partial N}{\partial \alpha_1} \qquad (3)$$

In case of running along α_1 solutions to the boundary value problem allow for the separation of variables

$$u = z_1 \sin(k\alpha_1 - \omega t); \quad v = z_2 \cos(k\alpha_1 - \omega t); \quad w = z_3 \cos(k\alpha_1 - \omega t);$$
$$\theta_2 = z_4 \cos(k\alpha_1 - \omega t); S = z_5 \sin(k\alpha_1 - \omega t); \; T_2 = z_6 \cos(k\alpha_1 - \omega t); \qquad (4)$$

$$\theta_2 = z_7 \cos(k\alpha_1 - \omega t); \; M_2 = z_8 \cos(k\alpha_1 - \omega t);$$

where $\omega = \omega_R + i\omega_I$ - complex natural frequency; κ- wave number, real value; ω_R - the real part of the complex frequency; ρ - density; $z_1(\alpha_2)(i = 1.8)$ - functions of the form of oscillations.

After substituting the relations (4) into equation (3), we have a spectral boundary value problem for the parameter ω For a system of eight ordinary differential equations with respect to a complex function of the form:

$$z_1 = z_5/\overline{A} + kz_2, \quad z_2 = z_6/\overline{C} + \upsilon k z_1 - k_2 z_3, \quad z_3 = -z_4 + k_2 z_2$$

$$z_4 = z_8/\overline{D} + \nu k^2 z_3, \qquad z_5 = h\left(\overline{E}k^2 - \rho\omega^2\right)z_1 + \nu h^2 z_6, \qquad (5)$$

$$z_6 = -h\rho\omega^2 z_2 - kz_5 - k_2 z_7, \qquad z_7 = -h\rho\omega^2 z_3 + \overline{E}/12h^3k^4 z_3 + \nu k^2 z_8 + k_2 z_6;$$
$$z_8 = z_7 + \overline{G}/3h^3k^2 z_4; \qquad z_5 = z_6 = z_7 = z_8 = 0; \qquad (\alpha_2 = 0, l)$$

E is expressed through operator elasticity moduli:

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$$\bar{E} = E\left[1-\Gamma_E^C(\omega_R)-i\Gamma_E^S(\omega_R)\right]\varphi.$$

As an example of a viscoelastic material, let's take three parametric relaxation nuclei. $R(t) = Ae^{-\beta t}/t^{1-\alpha}$, having a weak singularity.

For virtual work of inertia forces (δA_I) let's write the following relation:

$$\delta A_I = -\int_V \rho \ddot{u}_i \delta u_i dV,$$

Where ρ - body density; u_i – movement components; $\ddot{u}_i = \partial^2 ui / \partial t^2$; t - time.

Results and analysis. Consider a wedge-shaped plate, infinite along the axis. x_2. According to the Kirchhoff-Lyave hypotheses, we have:

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0;\ u_i = -x_3 \frac{\partial W}{\partial x_i};W\ (x_3)\quad W \qquad (6)$$

where W – deflection of the plate's median plane

From the set of solutions of the system, we will choose those that describe harmonic plane waves propagating along the axis. x_2

$$y_i = z_i(x_1)e^{i(\kappa x_2-\omega t)}. \qquad (7)$$

Substituting the solution (7) into the system of partial differential equations, we obtain a system of ordinary differential equations of the first order, solved with respect to the derivatives:

$$\begin{cases} z_1' = z_2 + \frac{z_n}{\chi h}; \\ z_2' = -\upsilon\kappa z_3 - \frac{6(1-\upsilon)}{3} z_5; \\ z_3' = \kappa z_2 - \frac{12}{h^3} z_6; \\ z_4' = \chi h\kappa z_3 + \kappa^2\left(\chi h - \frac{hc^2}{\Gamma_\text{Л}}\right) z_1; \\ z_5 = -\kappa z_6 + z_4 + \frac{h^3}{12\Gamma_\text{Л}}\omega^2 z_2; \\ z_6' = -\chi h\kappa z_1 - \left[\chi h + \frac{\kappa^2 h^3}{12\Gamma_\text{Л}}\left(2(1+\upsilon) - \frac{c^2}{\Gamma_\text{Л}}\right)\right] z_3 + \upsilon\kappa z_5. \end{cases} \qquad (8)$$

Figure 1 shows the variance curves of the phase velocities of the first three modes of vibrations in a Kirchhoff-Lyav plate with a linear law of thickness change.

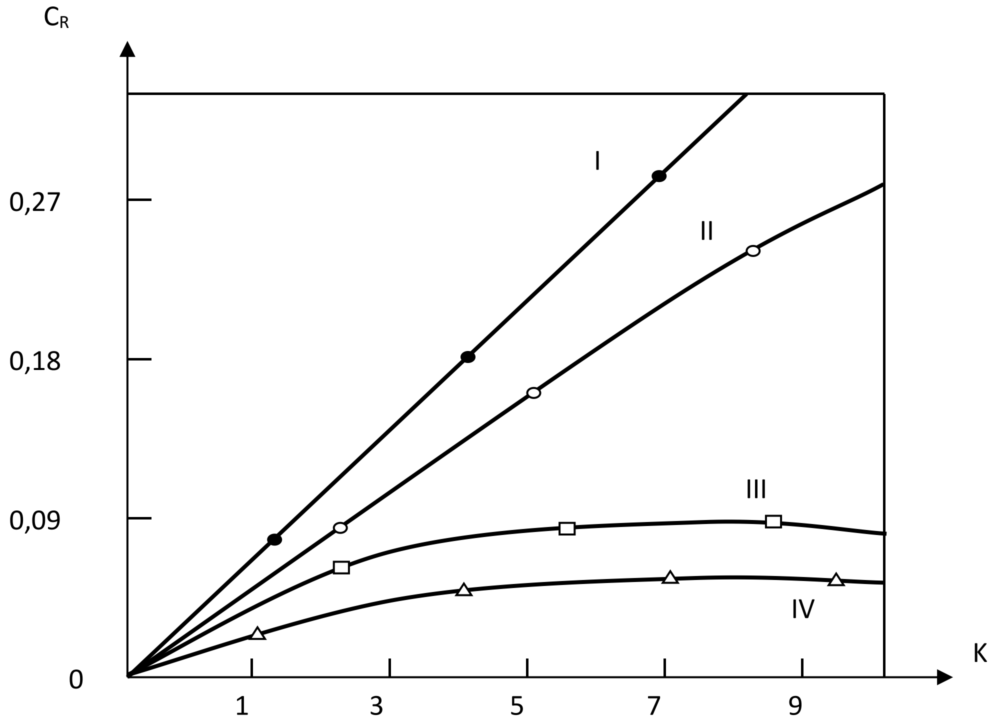

Fig.1. First-mode dispersion curves

I. $h_1=h_2=0,1$; II. $h_1=h_{2/2}=0,05$; III. $h_{2/100} = 0,001$; IV. $h_1=h_{2/1000}=0,001$

$h(x_1)=h_0 x_1^p,$ $\qquad o < x_1 \leq b,$

where the parameter p was taken as equal 1,5; 2; 2,5; 3 according to the designations of the curves 1, 2, 3, 4. At $p=1$, as noted above, the phase velocities asymptotically approach non-zero limit values, the first-mode curve increases monotonically.

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