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# MATHEMATICAL METHOD FOR DETERMINING THE VELOCITY OF LIQUIDS LEAKING FROM A VESSEL

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**Annotation:** The article examines the analysis of the process of liquid flowing out of the vessel opening using the mathematical modeling method. Based on Torricelli's law, the main equations for determining the flow rate and flow rate are presented. Based on the calculation results, the relationship between the liquid height and the outlet velocity was analyzed.
**Keywords**: Torricelli's Law, fluid flow, outflow velocity, mathematical modeling.
**Аннотация:** В статье рассматривается вопрос анализа процесса истечения жидкости из отверстия сосуда с помощью метода математического моделирования. На основе закона Торричелли приведены основные уравнения для определения скорости выхода и расхода потока. По результатам расчетов проанализирована зависимость между высотой жидкости и скоростью выхода.
**Ключевые слова:** Закон Торричелли, поток жидкости, скорость выхода, математическое моделирование.

**Introduction.** The process of liquid flowing out of a vessel is one of the important issues of hydrodynamics and has great importance in calculating processes in engineering, chemical, physical, and technical systems. Determining the flow rate of a liquid in a vessel is used in pump systems, reservoirs, fuel tanks, and industrial processes.

To explain this process, the theoretical foundations developed by Torricelli are used. According to Torricelli's law, the velocity of liquid outflow under gravity depends on the height of the liquid.

**Methods**. For the analysis of liquid flow, the law of conservation of energy and the law of conservation of mass are applied. Let us assume that a vessel with a known function $S=S(h)$ of the cross-sectional area S and height H is filled with liquid up to the level H. At the bottom of the vessel there is a hole with a surface area ω through which the liquid flows out. Let us consider the problem of finding the time t during which the liquid level decreases from the initial state H to an arbitrary state $h$, and the time T for the vessel to be completely emptied. To determine this, the rate of change of the liquid quantity (volume) in the vessel $\vartheta$ can be considered as a known function of the liquid level $h$ (pressure) in the vessel $\vartheta=\vartheta(h)$ (Figure 1).

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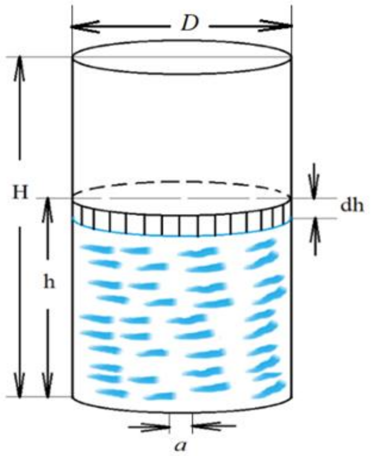


**Figure 1.** Depiction of the transition of a liquid in a reservoir from its initial state H to its state h.

Assuming that at some time t the height of the liquid in the vessel is equal to $h$, the amount of liquid $d\vartheta$ flowing out of the vessel in the time interval dt from t to t+dt can be calculated as the volume of a cylinder with a base area ω and a height $\vartheta(h)$. Therefore, we express dθ as a first-order differential equation with separable variables [3-6]

$$d\vartheta = w\vartheta(h)dt \qquad (1)$$

The indicated volume of liquid can be calculated in another way. Since the liquid flows out, the liquid level $h$ in the vessel decreases by a value of d(h), which can be expressed as a differential equation with separable first-order variables.

$$d\vartheta = -S(h)dh \qquad (dh<0) \qquad (2)$$

i.e., a negative sign $(dh<0)$was obtained, therefore for $d\vartheta$ from expressions (1) and (2) we write the following equation:

$$w\vartheta(h)dt = -S(h)dh \qquad (3)$$

This equation consists of a first-order equation with separable variables in the differential equations section of "Higher Mathematics" and its solutions are found as follows. [1-3]

The simplest first-order differential equation is an equation that is solved with respect to the derivative and does not contain the variable Y.

$$\frac{dy}{dx} = f(x) \qquad (4)$$

From the course of integral calculus, it is known that in this case, to find the unknown function Y, it is necessary to find the indefinite integral of the function f(x). General solution of equation (4)

$$dy = f(x)dx, \qquad \int dy = \int f(x)dx + C$$

or

$$y = \int f(x)dx + C \qquad (5)$$

The partial solution of equation (4) with the initial condition $y\|_{x=x0} = y_0$ is often convenient to write as a definite integral. Indeed, we can represent an initial function as a definite integral with a given lower boundary and a variable upper boundary, for example

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$$y=\int_{x_0}^{x} f(t)dt+C$$

can be written in the form: at $x=x_0$ this integral becomes zero, therefore, to fulfill the initial conditions, it is necessary to take $s=y_0$, consequently, the particular solution (1)

$$y=y_0+\int_{x_0}^{x} f(t)dt$$

can be written as. You can calculate the value of C and find the particular solution. Based on this, we can write (3) as follows

$$\frac{dt}{dh}=\frac{S(h)}{w\vartheta(h)}$$

Select the variables:

$$\int dt=-\int \frac{S(h)}{w\vartheta(h)}dh$$

Integrating from this

$$t=-\frac{1}{w}\int_{H}^{h}\frac{S(h)}{\vartheta(h)}dh=\frac{1}{w}\int_{H}^{h}\frac{S(h)}{\vartheta(h)}dh \quad (6)$$

When integrating, when changing the boundaries, the sign of the integral changes to the opposite. Since when the vessel is completely emptied, $h=0$, then the time of complete emptiement of the vessel T is found by the formula.

$$T=\frac{1}{w}\int_{0}^{H}\frac{S(h)}{\vartheta(h)}dh \quad (7)$$

If the liquid flows out of a small hole or short tube, then it is determined by Torricelli's law [1].

$$\vartheta=\mu\sqrt{2gh} \quad (8)$$

Here, g− is the gravitational acceleration, μ− is the empirical coefficient (flow coefficient). Taking this into account (8), we will find the formulas (7) and (8) for the flow rate t and the time of complete emptying of the liquid in the vessel T.

$$t=\frac{1}{w\mu\sqrt{2g}}\int_{h}^{H}\frac{S(h)}{\sqrt{h}}d\eta \quad (9)$$

and

$$T=\frac{1}{w\mu\sqrt{2g}}\int_{0}^{H}\frac{S(h)}{\sqrt{h}}d\eta \quad (10)$$

Using definite integrals, we find t and T, i.e., the velocity of the liquid out of the vessel t and the time it takes for the liquid to be completely emptied in the vessel T. Applying this theory, we solve this problem.

**Results**. A railway tank with length L and diameter D is filled with kerosene. Let's determine the discharge time of the tank when the kerosene is drained through a short discharge pipe (valve) located under the tank and having a cross-sectional area w. (Figure 2).

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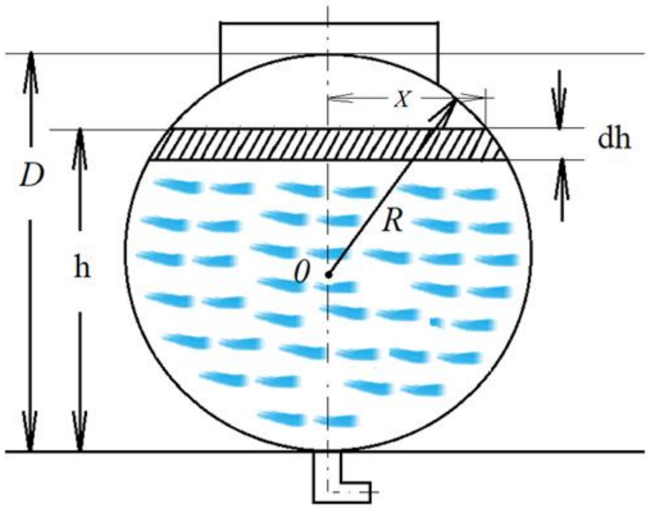


**Figure 2.** Image of the cross-section of a kerosene tank.

Solution: The surface area of the petroleum product S (h) is a variable quantity and is determined by the following formula. [4-9]

$$S(h)=2*L=2L\sqrt{R^2-(h-R^2)}=2L\sqrt{D(D-h)h}$$

Therefore, substituting S (h) into formula (10), we obtain the following.

$$T=\frac{2L}{w\mu}\int_0^D \frac{\sqrt{(D-h)h}}{\sqrt{h}}dh=\frac{4LD\sqrt{D}}{3w\mu\sqrt{2g}}$$

In particular, if $L=12m$, $D=2.6m$, $w=0.01m^2$ and the consumption coefficient $\mu=0.6$ (for kerosene),

$$T=\frac{4*12*2.6\sqrt{2.6}}{3*0.01*0.6\sqrt{19.62}}=2520\ sek=40\ min$$

**Discussion.** The obtained results fully correspond to Torricelli's law. Both theoretically and experimentally, it has been confirmed that with increasing altitude, the gravitational energy increases and the exit velocity increases.

On the other hand, if the current coefficient is not taken into account, the results reflect the ideal state, and a certain error is observed in practice. Therefore, empirical coefficients should always be introduced in engineering calculations.

**Conclusion**: In conclusion, when gas and liquid flow slowly, their layers are arranged parallel to each other. Such a flow is called laminar. If gas and liquid flow rapidly, their layers mix. Such a flow is called turbulent. The velocity of the liquid flowing out of the vessel is determined by the expression $\vartheta=\sqrt{2gh}$ according to Torricelli's law. This method can be used to optimize hydraulic calculations in various technical systems.

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