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CONSTRUCTION OF ICOSAHEDRAL SIERPINSKI FRACTALS BASED ON ARITHMETIC BINOMIAL POLYNOMIAL PROPERTIES

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Abstract: This paper presents a mathematically grounded approach to constructing 3D icosahedral Sierpinski fractals using the arithmetic properties of binomial polynomials. The study integrates combinatorial binomial structures, classical fractal theory, and modern geometric transformations to generate iterative fractal models with increasing structural complexity.

Keywords: Fractal, binomial structures, sierpinski, Pascal’s triangle, icosahedral, geometric modeling.

Introduction: Fractals represent a class of geometrical objects characterized by self-similarity, recursive structure, and scale invariance. They naturally arise from the arithmetic properties of binomial coefficients, specifically from the parity distribution in Pascal’s triangle, which forms the classical Sierpinski triangle. When extended to three dimensions, these structures appear in technological fields such as antenna engineering, computer graphics, and structural modeling [1].

Binomial-based fractals provide an elegant connection between discrete mathematics and geometric modeling. Their recursive nature makes them suitable for constructing complex shapes using simple iterative rules. The aim of this study is to:

- formalize a method for constructing a 3D icosahedral Sierpinski fractal using binomial polynomial arithmetic;
- describe the algorithmic methodology for generating multiple iterations;
- highlight their relevance to contemporary scientific and engineering applications.

Literature Review: Bondarenko, Krinchik, Sierpinski and other classical authors describe how the modular–arithmetic properties of binomial coefficients generate fractal structures. In particular, the binomial expansion

$$(1+1)^n = \sum_{k=0}^{n} C(n,k) \qquad (1)$$

and its mod-2 form

$$C(n,k) \bmod 2 \qquad (2)$$

constitute the mathematical basis of the Sierpiński triangle pattern [1], [3].

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Bondarenko (1983) provides a detailed explanation of the connection between fractal patterns and multi-faceted geometric structures, which today serves as a direct theoretical foundation for constructing contemporary 3D fractals.

In recent years, fractal structures have been actively used in the following areas:

Fractal antennas are notable for:

- broadband performance,
- compact physical size,
- multi-band operation.

Studies published after 2019 confirm the growing use of models such as the Menger sponge, Sierpinski gasket, Koch curve and Cantor fractals in antenna design [4–10].

Video graphics and 3D animation. Modern VFX systems such as Blender, Houdini, Cinema4D and Maya widely use IFS-based fractal generation for:

- procedural geometry,
- organic surface creation,
- sci-fi modelling,
- real-time volumetric rendering [11], [12].

Nanostructures, metamaterials and electromagnetic waves. Fractal surfaces enable multi-scale electromagnetic scattering and wave shaping. Each iteration adds additional resonance frequencies, contributing to improved EM response in advanced materials [13–16].

Medical Imaging and Biological Modelling. Last years fractal geometry has been applied to modelling:

- vascular networks,
- alveolar structures,
- neural tissue architectures [17–18].

Compression and deep-learning models. Fractal coding and fractal neural networks are increasingly used in next-generation signal and image compression technologies [19].

Methodology

Constructing the Icosahedron. The vertices of a regular icosahedron are defined as:

$$\varphi=\frac{1+\sqrt{5}}{2} \qquad (3)$$

$$V=\{(\pm1,\pm\varphi,0),(0,\pm1,\pm\varphi),(\pm\varphi,0,\pm1) \qquad (4)$$

Normalization:

$$V_i=\frac{V_i}{\|V_i\|} \qquad (5)$$

ensures that the polyhedron lies on a unit sphere.

Applying Sierpinski iterations to each triangular face. Given a triangular face with vertices A, B, C:

First iteration:

$$AB=\frac{A+B}{2},BC=\frac{B+C}{2},CA=\frac{C+A}{2} \qquad (6)$$

Second iteration produces three subtriangles:

$$\Delta(A,AB,CA),\Delta(AB,B,BC),\Delta(CA,BC,C) \qquad (7)$$

Third and fourth iterations follow the same recursive rule:

$$S(n+1)=3\cdot S(n) \qquad (8)$$

Forming the 3D fractal surface. The 3D fractal geometry is generated through Iterated Function Systems (IFS):

$$T_i(x)=\frac{1}{2}(x-v_i)+v_i,i=1,2,3 \qquad (9)$$

These transformations represent the generalized form of a 3D Sierpinski tetrahedron. When applied to each face of the icosahedron, they produce a recursively carved fractal surface.

4. Results

This study constructs 2nd-, 3rd-, and 4th-iteration 3D Sierpinski Icosahedron models.

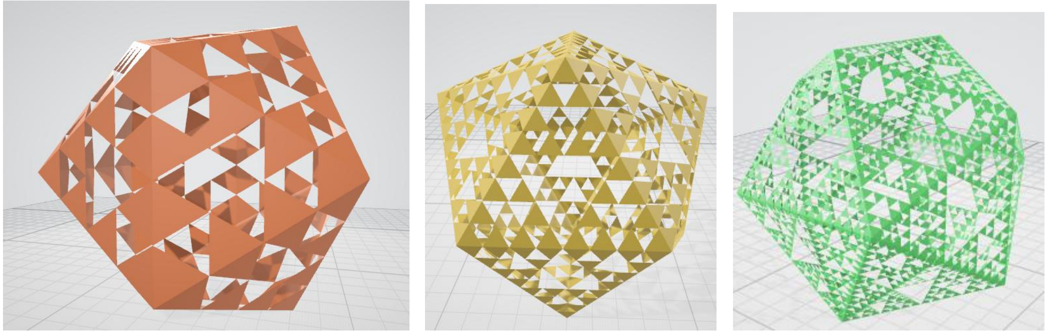

Fig. 1. 2nd-, 3rd-, and 4th-iteration 3D Sierpinski Icosahedron models.

5. Conclusion

The arithmetic properties of binomial polynomials provide a precise and elegant foundation for modelling fractal structures.

By applying Sierpinski iterations to the faces of a regular icosahedron, the resulting 3D models demonstrate significant potential in video graphics and antenna engineering. Such multi-scale geometries expand resonance bandwidth in antennas and provide high-detail procedural structures in modern computer graphics.

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