

Forecasting the stage of development of scientific work on determining the hardness of construction materials in accordance with international standards by the least square method.

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Abstract: This article is devoted to forecasting the stage of development of scientific work on determining the hardness of construction materials in accordance with international standards using the least square method. The article focuses on the history, significance and mathematical models of the method of least squares. One of the important points of the article is that we can achieve possible indicators for the coming years based on statistical data.

Key words: International standards, hardness, least squares method, mathematical modeling, forecasting.

The method of least squares originated in the fields of astronomy and geodesy as scientists and mathematicians sought to solve the problems of navigating the Earth's oceans during the Age of Discovery. Accurate description of the behavior of celestial bodies became the key to ships sailing on the open seas, where sailors no longer relied on land observations for navigation.

The method was the culmination of several advances that occurred during the eighteenth century:[6]

A combination of different observations as the best estimate of the true value; errors, perhaps first expressed by Roger Cotes in 1722, decrease by addition instead of multiplication.

A combination of different observations taken under the same conditions is different from doing one's best to accurately observe and record a single observation. The approach is known as the method of averages. This approach was used in particular by Tobias Mayer in 1750 to study the disintegration of the Moon and by Pierre-Simon Laplace in 1788 to explain the differences in the motions of Jupiter and Saturn[1].

A combination of different observations taken under different conditions. The method was recognized as the least absolute deviation method. This was notably done by Roger Joseph Boskovich in 1757 in his work on the shape of the Earth, and by Pierre-Simon Laplace in 1789 and 1799 for the same problem.

Develop a criterion that can be evaluated to determine when a solution with minimum error has been reached[3]. Laplace tried to determine the mathematical form of the probability density for errors and determined an estimation method that minimizes the estimation error. To this end, Laplace used a symmetric double-exponential distribution, which we now call the Laplace distribution, to model the error distribution, and used the sum of the absolute deviation as the

estimation error. He considered these to be the simplest guesses he could make, and he hoped to take the arithmetic mean as his best estimate. Instead, its estimator was the posterior median[2].

The first clear and concise exposition of the method of least squares was published by Legendre in 1805.[7] The technique is described as an algebraic procedure for fitting linear equations to data, and Legendre demonstrates a new method by analyzing the same data as Laplace for the shape of the Earth. A decade after Legendre's publication, the method of least squares was adopted as a standard tool in astronomy and geodesy in France, Italy, and Prussia, representing an extraordinarily rapid adoption of the scientific technique.

Statistical data for forecasting the stage of development of scientific work on determining the hardness of construction materials in accordance with international standards by the least square method were obtained from the Scopus database [5]. According to it, the number of scientific works conducted and published between 2014 and 2024 was sorted by year.

№	Years	Results
1	2014-y	4
2	2015-y	6
3	2016-y	8
4	2017-y	10
5	2018-y	22
6	2019-y	20
7	2020-y	20
8	2021-y	25
9	2022-y	20
10	2023-y	23
11	2024-y	20

Table 1. Information from the Scopus database.

Forecasting using the method of least squares. Using the method of least squares, the desired curve is found using the combined formulas for the point of intersection of the arm with the Y reading and the tangent of the angle produced by the X reading[3]:



$$b_1 = \frac{n \sum_{i=1}^n X_i Y_i - (\sum_{i=1}^n X_i)(\sum_{i=1}^n Y_i)}{n \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2}; \quad (1)$$

$$b_0 = \frac{\sum_{i=1}^n Y_i}{n} - b_1 \frac{\sum_{i=1}^n X_i}{n}, \quad (2)$$

where n is the sample size, X_i is the observation in the i -step, Y_i is the value of the observation in the i -step [4].

Therefore, the equation of the regression closure will look like this:

$$\vec{Y} = b_0 + b_1 X. \quad (3)$$

The reliability coefficient is

$$R^2 = 1 - \frac{\sum_{i=1}^n (Y_i - \vec{Y}_i)^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \quad (4)$$

It is calculated by the formula [5]. Here, \vec{Y}_i is the value of the observation in step i , \bar{Y} is the value of equation (3) in step i , \bar{Y} is the average value of Y -observations, i.e[6].

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n} \quad (5)$$

Using the information in Table 1, we fill in Table 2.

$$b_1 = \frac{11 * 1280 - 66 * 178}{11 * 506 - 66^2} = 1.927$$

$$b_0 = \frac{178}{11} + 1.927 * \frac{66}{11} = 4.618$$

Years	X	simple	b_1	b_0
2014-y	1	4	1.927	4.618
2015-y	2	6	1.927	4.618
2016-y	3	8	1.927	4.618
2017-y	4	10	1.927	4.618
2018-y	5	22	1.927	4.618
2019-y	6	20	1.927	4.618
2020-y	7	20	1.927	4.618
2021-y	8	25	1.927	4.618
2022-y	9	20	1.927	4.618
2023-y	10	23	1.927	4.618
2024-y	11	20	1.927	4.618
	11			
Sum	66	178		

$X_i * Y_i$	X_i^2	\bar{Y}_i	$(Y_i - \bar{Y}_i)^2$	$(Y_i - \bar{Y})^2$	R^2	\bar{Y}
4	1	6.55	6.48	148.397	0.7123	16.182
12	4	8.47	6.11	103.669		16.182
24	9	10.40	5.76	66.942		16.182
40	16	12.33	5.42	38.215		16.182
110	25	14.25	59.99	33.851		16.182
120	36	16.18	14.58	14.579		16.182
140	49	18.11	3.58	14.579		16.182
200	64	20.04	24.64	77.760		16.182
180	81	21.96	3.86	14.579		16.182
230	100	23.89	0.79	46.488		16.182
220	121	25.82	33.85	14.579		16.182
1280	506	178	165.05	573.636		

Table 2. Data processing based on mathematical models.

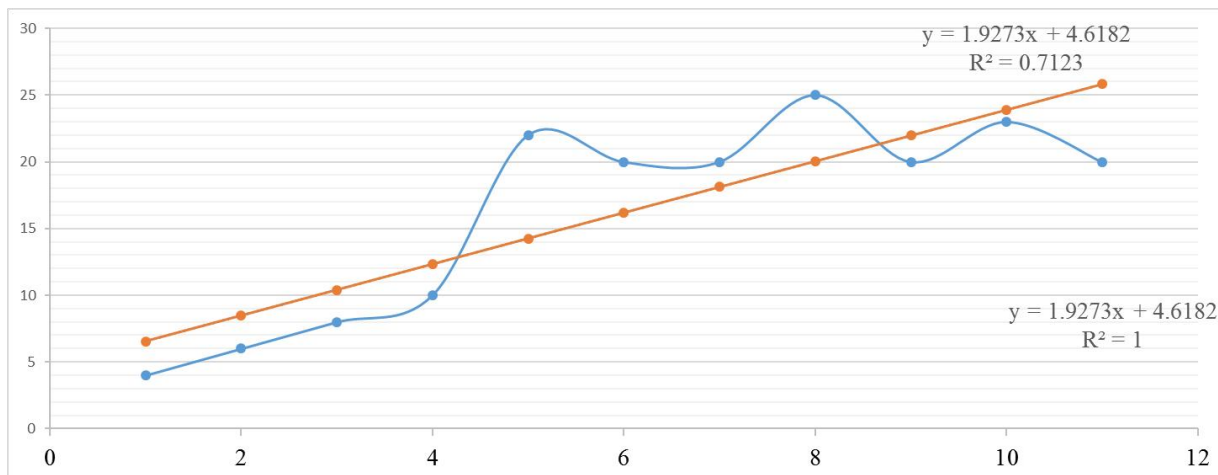


Fig. 1. Visual representation of data with a regression line.

The found values (3) are brought to the regression straight line equation and the equation in the form of $\vec{Y}_i = b_0 + b_1 * X = 4.618 + 1.927 * x_i$ is generated. The value $x=11$ is put into the equation of the regression line and the number $y=27.75$ is generated.

Year	X	forecasting for the coming years	\vec{Y}_i	b_1	b_0
2014-y	1	4	6.55		
2015-y	2	6	8.47		
2016-y	3	8	10.40		
2017-y	4	10	12.33		
2018-y	5	22	14.25		
2019-y	6	20	16.18		
2020-y	7	20	18.11		
2021-y	8	25	20.04		
2022-y	9	20	21.96		
2023-y	10	23	23.89		
2024-y	11	20	25.82		
2025-y	12		27.75		
2026-y	13		29.67	1.93	4.62
2027-y	14		31.60	1.93	4.62
2028-y	15		33.53	1.93	4.62
2029-y	16		35.45	1.93	4.62

2030-y	17		37.38	1.93	4.62
2031-y	18		39.31	1.93	4.62
2032-y	19		41.24	1.93	4.62
2033-y	20		43.16	1.93	4.62

Table 3. A forecast prepared for the coming years.

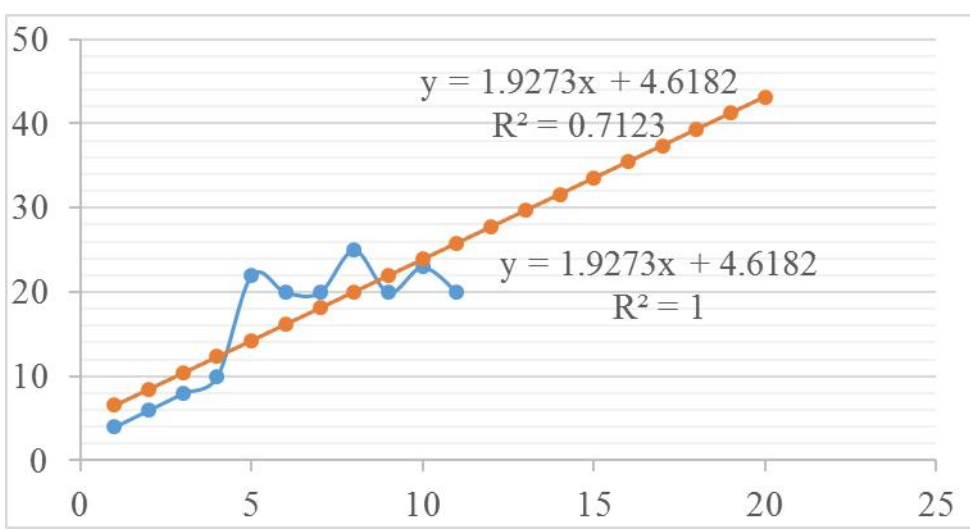


Fig. 2. A visual representation of expected results in the coming years.

Conclusion.

Hardness measurement is one of the most common criteria used in acceptance methods in the steel industry. In addition, microhardness methods are used as control variables in quality assurance in the microelectronics industry. Customers of these SRMs include the industries that presently require traceability to a National Metrology Institution, secondary standard producers, instrument manufacturers, and end users as well[7].

We can see from Fig. 2 that the number of scientific works on the standards for assessing the stiffness of construction materials in the Scopus database by 2033 with a reliability coefficient of 71% can reach 43.

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