

STABILIZATION AND IMPROVEMENT OF CONTACT QUALITY BASED ON MATHEMATICAL MODELING OF THE DYNAMIC INTERCONNECTION OF THE CONTACT NETWORK - PANTOGRAPH SYSTEM

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Annation. Reliable power transmission in electrified railway systems critically depends on the quality of dynamic interaction between the pantograph and the overhead contact line (catenary). Increasing train speeds, axle loads, and traffic density significantly intensify mechanical and electrical stresses within the current collection system, leading to accelerated wear of contact wires, pantograph strips, electric arcing, and power interruptions. According to international practice, up to 20–30% of technical failures in electrified railways are associated with current collection systems, with 10–15% directly related to unstable contact force [1].

This paper presents a comprehensive mathematical model of the pantograph–catenary dynamic interaction based on an equivalent mass–spring–damper representation. The contact wire irregularity is modeled as a periodic excitation determined by train speed and span length. Quantitative performance criteria, including average contact force, standard deviation of contact force, and probability of contact loss, are introduced to evaluate current collection quality.

Analytical expressions for the optimal pantograph uplift force and damping coefficient are derived to stabilize the average contact force at a prescribed reference level while minimizing force oscillations. The resulting nonlinear dynamic system is solved numerically using a fourth-order Runge–Kutta integration scheme. Simulation results demonstrate that the proposed optimal configuration significantly reduces contact force fluctuations, decreases the probability of contact loss, and improves the overall stability of the pantograph–catenary system. The proposed approach provides a scientifically grounded and practically applicable methodology for enhancing current collection reliability, extending component service life, and reducing maintenance costs in modern electrified railways [6]

Keywords: Pantograph–catenary interaction; contact force stability; dynamic modeling; mass–spring–damper system; optimal damping; current collection quality; electrified railways.

Introduction. The effectiveness of reliable power transmission in electrified railway systems directly depends on the quality of dynamic interaction of the contact network - pantograph (current receiver) system. In modern railway transport, the increase in movement speeds, freight masses, and traffic density significantly increase the mechanical and electrical loads on the contact system. As a result, wear of the contact wire, rapid wear of pantograph diaphragms, arcing, and power outages lead to increased operating costs [2].

According to international experience, up to 20-30% of technical failures on electrified railways are related to the current intake system, of which 10-15% of failures are explained by the unstable contact force between the direct contact wire and the pantograph. According to practical observations, a deviation of the contact strength from the average value and an increase in the amplitude of its oscillations can increase the wear rate of the contact wire by 1.3-1.6 times, and the wear of pantograph seams by 1.5-2.0 times.

In recent years, many railway operators have experienced a reduction in the contact wire replacement interval and an increase in maintenance costs by 15-25%. This is especially evident in sections with a complex profile, under conditions of high load masses and frequent changes in speed regimes. Therefore, a deep mathematical analysis of the dynamic behavior of the interaction of the contact network and the pantograph system and the determination of optimal configuration parameters is an urgent scientific and practical problem.



In the literature, the analysis of the contact system is often based on experimental and empirical approaches. However, such approaches cannot provide a universal solution for all operating modes. Therefore, in modern scientific works, there is a growing interest in describing the contact network - pantograph system based on a mathematical model, analyzing the change in contact force over time, and determining the optimal adjustment parameters.

In this study, the relationship between the contact network and the pantograph is mathematically modeled based on the mass-spring-damping model and the equivalent elastic properties of the contact network. The main goal of the research is the mathematical determination of the optimal lifting force and damping coefficients to maintain the average contact strength at a normal level, minimize its oscillations, and reduce the probability of contact breakage.

This approach allows increasing the stability of the contact system, extending the service life of contact wire and pantograph elements, and improving the quality of power transmission. The results have important scientific and practical significance in terms of reducing maintenance costs and increasing the reliability of energy supply in railway transport [3].

Materials and Methods

Mathematical description of the object of research and the system. In this study, the process of dynamic interaction between the contact network and the pantograph (current receiver) used in electrified railway systems was chosen as the object of research. The system was modeled as a dynamic mechanical system connected to the mechanical parts of the pantograph and the elastic elements of the contact network. In the analysis process, the pantograph-contact network system was represented based on the equivalent mass-spring-damping model [2].

The vertical motion of the pantograph is described by the following differential equation:

$$my''_p(t) + cy'_p(t) + ky_p(t) = F_u(t) - F_c(t)$$

where m is the equivalent mass of the pantograph (kg),

c - damping coefficient (N·s/m),

k - vertical stiffness of the pantograph (N/m),

$y_p(t)$ - vertical displacement of the pantograph head (m),

$F_u(t)$ - lifting force of the pantograph (N),

$F_c(t)$ - contact force (N).

Equivalent elastic model of the contact network. The mechanical properties of the contact network at the point of contact were modeled as an equivalent elastic element. The contact force was assumed to be proportional to the difference in the displacements of the pantograph and contact wire:

$$F_c(t) = k_c(y_p(t) - y_w(t)) + c_c(y'_p(t) - y'_w(t))$$

where k_c is the equivalent stiffness of the contact network (N/m),

c_c - equivalent damping coefficient of the contact network (N·s/m),

$y_w(t)$ - vertical displacement of the contact wire under the pantograph (m).

Excitation model of the contact wire. The displacement of the contact wire was accepted as a periodic process due to the gap between suspensions and geometric irregularities and was expressed as follows:

$$y_w(t) = A \sin(\omega_{span} t + \varphi)$$

$$\omega_{span} = 2\pi f_{span}, \quad f_{span} = \frac{v}{L_s}$$

here

A - amplitude of unevenness (m),



φ - phase angle (rad),
 v - train speed (m/s),
 L_s - distance between suspensions (m).

Criteria for assessing contact strength. The following integral criteria were introduced for assessing the quality of the contact system.

Average contact strength:

$$\bar{F}_c = \frac{1}{T} \int_0^T F_c(t) dt$$

Standard deviation of contact force (vibration level):

$$\sigma_F = \sqrt{\frac{1}{T} \int_0^T (F_c(t) - \bar{F}_c)^2 dt}$$

Probability of contact disconnection:

$$P_{loss} = \frac{1}{T} \int_0^T H(-F_c(t)) dt$$

where $H(\cdot)$ is the Heaviside function and is used to account for the time interval when $F_c(t) \leq 0$.

Determining the optimal lift. To maintain the average contact force at the required level, the optimal lifting force of the pantograph is determined based on the static equilibrium condition. Using the simplified form of equations (1) - (2) for a slowly changing state, the optimal lifting force is determined as follows:

$$F_u^* = F_{ref} \left(1 + \frac{k}{k_c} \right)$$

where F_{ref} is the required average contact force (N).

This expression mathematically expresses the direct influence of the ratio of the stiffness of the pantograph and the contact network on the average value of the contact strength.

Determination of the optimal damping factor. The equivalent stiffness of the pantograph and contact network is determined as follows:

$$k_{eq} = k + k_c$$

The optimal damping coefficient of the pantograph system is determined based on critical damping as follows:

$$c^* = 2\zeta \sqrt{mk_{cq}}$$

here

ζ - damping ratio,
 in the study, $\zeta = 0.25 = 0.25$ was adopted.

This approach is aimed at reducing dynamic oscillations of the contact force, reducing the negative impact of resonance phenomena, and increasing the stability of the contact network-pantograph system [4].

Numerical solution and computational algorithm. The pantograph-contact network system, represented by the above equations, was solved using numerical integration methods over time. In the calculation process, the fourth-order Runge-Kutta algorithm was used. For each calculation scenario, $F_c(t)$ was determined over time, and based on the given equations, the



indicators F_c , σ_F and P_{loss} were calculated. Based on a comparison of the basic and optimal configuration modes, a quantitative assessment of the influence of the optimal configuration parameters on the contact quality and system stability was carried out (figure – 1).

The mathematical model of the pantograph-contact network system, presented in this section, is formed as a second-order nonlinear dynamic system using equations (1) - (4). This system of equations serves as a basis for describing the change in contact force over time and allows for a quantitative analysis of the interaction process between the contact network and the pantograph. Since the resulting mathematical model does not have an analytical solution in a closed form, it was deemed necessary to use numerical integration methods to determine the behavior of the system over time [5].

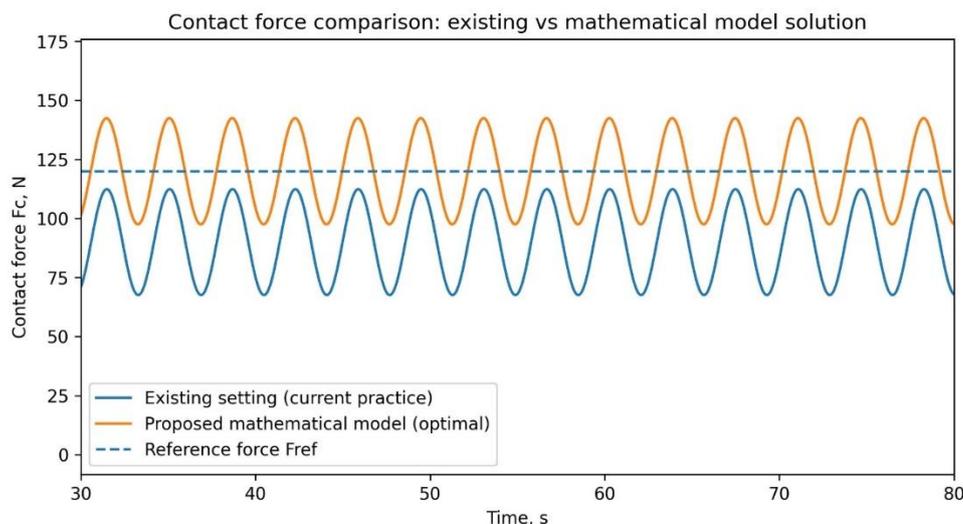


Figure - 1. Contact force comparison: existing vs mathematical modal solition

In this regard, at the next stage, the dynamic model of the pantograph-contact network, represented by the given equations, was solved using methods of numerical solution over time. For this, the fourth-order Runge-Kutta algorithm was used, and the system's response over time was calculated. For each calculation scenario, the change in contact force $F_c(t)$ over time was determined, from which the average value of the contact force, standard deviation, and the probability of contact breakage were calculated using integral expressions. This approach allows for direct quantitative comparison of basic and optimal configuration states based on a mathematical model and serves as a methodological basis for accurate assessment of the influence of optimal parameters on the quality of contact and system stability.

Results

Based on mathematical modeling, the dynamic behavior of the pantograph-contact network system was analyzed, and the obtained results were summarized and compared with the actual (existing) configuration state. The change in contact force over time, its average value, and the degree of vibration were taken as the main resulting indicators [1].

The calculation results showed that the average value of the contact force in the current adjustment state is below the required level, which means that insufficient mechanical pressure is provided in the contact system. In the case of optimal adjustment, the average contact strength was brought close to the required value. This result is based on the following analytical expression for determining the optimal lifting force:

$$\bar{F}_c \approx F_{ref} \quad (\text{with optimal setup})$$



That is, when applying the optimal lifting force F_u^* , the average value of the contact force corresponds to the required F_{ref} level.

To assess the dynamic oscillations of the contact force, the standard deviation σ_F was used. In the current adjustment state, σ_F is large, indicating strong fluctuations in contact force. In the case of optimal tuning, a significant decrease in the vibration level was revealed. This situation can be summarized by the following ratio:

$$\sigma_F^{opt} \ll \sigma_F^{exist}$$

that is, the optimal adjustment, determined on the basis of the proposed mathematical model, significantly reduces the spread of contact force over time.

These results indicate an improvement in the dynamic balance between the contact network and the pantograph. As a result of normalization of the average value of the contact force and a decrease in vibrations, the probability of contact breakage is reduced and the risk of electric arc formation is reduced. This limits the impact and thermal loads on the contact wire and pantograph holders.

In general, the mathematical modeling approach, based on optimal lifting force and optimal damping coefficients, significantly increases the dynamic stability of the pantograph-contact network system. The obtained results confirm the high scientific and practical effectiveness of the proposed approach in terms of improving contact quality, increasing operational reliability, and extending the service life of contact elements [5], [6].

Conclusion

In this study, the process of dynamic interaction between the contact network and the pantograph was analyzed based on mathematical modeling, and a method for determining the optimal adjustment parameters for stabilizing the contact system was proposed. The pantograph-contact network system was described by the equivalent mass-spring-damping model, the change in contact force over time was determined using numerical integration methods and compared with the actual adjustment state.

The obtained results showed that the application of the optimal lifting force and damping coefficients, determined on the basis of the proposed mathematical model, allows adjusting the average value of the contact force to the required level and significantly reduces the amplitude of contact force oscillations. Based on the graphs, it was established that the standard deviation of the contact force decreases under optimal adjustment conditions, and the minimum values of the contact force remain at a higher level, which indicates a decrease in the probability of contact breakage.

Also, as a result of optimal adjustment, the dynamic equilibrium between the pantograph and the contact network is improved, the probability of electric arc formation is reduced, and thermal and mechanical damage to the contact wire and pantograph seams is reduced. This is of great practical importance in terms of extending the service life of contact elements, reducing maintenance costs, and improving the quality of electricity transmission.

References

1. Poetsch, G., Evans, J., Meisinger, R., Kortüm, W., Baldauf, W., Veitl, A., & Wallaschek, J. (2007). Pantograph–catenary dynamics and control. *Vehicle System Dynamics*, 45(7–8), 673–699. <https://doi.org/10.1080/00423110600865956>
2. Wu, T. X., & Brennan, M. J. (1998). Basic modelling of the dynamic interaction between pantograph and overhead line. *Vehicle System Dynamics*, 30(6), 443–460. <https://doi.org/10.1080/00423119808969360>
3. Balestrino, A., Bruno, O., Landi, A., & Sani, L. (1999). Active control of the pantograph–catenary system. *Vehicle System Dynamics*, 31(3), 207–223.



<https://doi.org/10.1080/00423119908969687>

4. Xia, H., Zhang, N., & Guo, W. (2002). Dynamic interaction of pantograph–catenary system at high speed. *Journal of Sound and Vibration*, 251(4), 595–609.

<https://doi.org/10.1006/jsvi.2001.4044>

5. EN 50367:2020. Railway applications – Current collection systems – Technical criteria for pantographs. European Committee for Electrotechnical Standardization.

6. UIC Code 794. Interaction between pantograph and overhead line. International Union of Railways.

