eISSN: 2229-3213 pISSN: 2229-3205
The Journal of Applied Science and Social Science
Volume 16,Issue 02 February, 2026

AN INTEGRATED APPROACH THAT COMBINES TRIZ, HEURISTIC LEARNING, AND MATHEMATICAL MODELING TO DEVELOP PRESCHOOLERS' MATHEMATICAL THINKING—FROM SETS AND QUANTITIES TO NUMBERS AND RELATIONSHIPS

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ABSTRACT

Modern preschool education enables teachers to design authorial programs for children's mathematical development. However, sustainable innovation requires a strong methodological grounding in mathematical theory and in evidence-based approaches used in both preschool and early primary curricula. This article substantiates the pedagogical potential of integrating the Theory of Inventive Problem Solving (TRIZ), heuristic learning, and mathematical modeling in preschool mathematics. The paper outlines a functional definition of mathematical modeling with preschoolers, proposes a classification of modeling practices (by set-theoretic meaning and spatial orientation), and presents a developmental logic linking sensory, intellectual, and creative abilities across preschool ages.

KEYWORDS

preschool education; early mathematics; mathematical modeling; TRIZ; heuristic learning; logical–schematic thinking; set theory; quantities; didactic games; tangram; continuity preschool–primary

INTRODUCTION

Contemporary early childhood education offers broad opportunities for educators to create original programs that foster preschoolers' mathematical development. Yet curricular creativity alone does not guarantee quality. Meaningful program design presupposes a deep understanding of mathematical principles, methodological approaches to concept formation, and the continuity requirements embedded in modern preschool and early primary curricula.

The relevance of this topic is driven by three major factors. First, early mathematics is increasingly understood not as the memorization of isolated facts but as the development of cognitive tools: reasoning, modeling, classification, comparison, and control of one's own learning results. Second, modern educational standards emphasize humanistic principles: learning through play, social interaction, and supportive adult–child collaboration while respecting individual pace and capability. Third, the family environment is recognized as a powerful context for mathematical development, and educators need practical, transferable strategies that can extend beyond the classroom.

In this context, the adaptation of TRIZ-based problem solving, heuristic learning, and mathematical modeling to the specificity of children's creative activity appears particularly promising. These approaches enrich the formation of mathematical concepts, provide a foundation for refining curriculum requirements, and offer actionable ideas for both institutional and family-based learning.

AIM, TASKS, OBJECT OF RESEARCH

Aim

To theoretically justify and methodologically describe how TRIZ, heuristic learning, and mathematical modeling can be integrated to strengthen preschoolers' mathematical development within the content sequence "Set → Quantity → Number → Relationship."

Tasks

1. To analyze contemporary approaches to preschool mathematical development and clarify the role of modeling, heuristics, and TRIZ in content renewal.
2. To define mathematical modeling for preschoolers and propose a practical classification of modeling techniques.
3. To describe the developmental logic of sensory, intellectual, and creative abilities in modeling activity across preschool ages.
4. To outline diagnostic task types that support continuity between preschool and early primary mathematics.
5. To justify the sequence "Set → Quantity → Number → Relationship" as a methodological backbone for structuring content.

Object of research

The process of mathematical development of preschool children within preschool educational settings and family upbringing.

(Optional, if your journal requires it:)

Subject of research

Methods and technologies for forming preschoolers' mathematical representations through TRIZ, heuristic learning, and mathematical modeling.

5. LITERATURE REVIEW

Literature Review (with citations)

Research in developmental psychology and preschool pedagogy consistently frames the preschool period as a **sensitive stage** for cognitive development, when the foundations of logical thinking, imagination, classification, generalization, and self-regulation are actively formed. In the cultural-historical tradition, children's higher mental functions are viewed as developing through **social mediation** and the internalization of culturally shaped tools, with language and sign systems playing a central role in the transition from external action to inner regulation and reasoning [1], [2]. Complementary perspectives on early cognitive development emphasize that preschool thinking remains closely connected to action and imagery, while progressively forming more stable mental structures that later support logical operations [4]. Within activity-oriented developmental frameworks, play is not treated as merely recreational, but as the **leading context** through which children appropriate social experience, internalize rules and roles, and build symbolic meanings—processes essential for the emergence of voluntary control and reflective regulation [5].

A substantial body of methodological research links the development of higher mental functions to children's mastery of **means (tools) that reveal essential relations and patterns**. The stepwise formation of mental actions suggests that children acquire stable intellectual operations when adults organize purposeful activity that moves from material actions to verbal guidance and then to internal mental operations [6]. This theoretical line supports the idea that mathematical education in preschool should be built on structured activity that gradually shifts children from direct manipulation toward conceptual control. In this paradigm, **mathematical modeling** is frequently treated as a special cognitive instrument enabling an orderly transition from concrete operations to mediated representations: substitutions, schemata, and conditional symbols [6], [8]. Studies of preschool abilities underline the developmental value of visual modeling actions and the gradual expansion of children's representational repertoire, which supports more complex cognitive operations and reflective monitoring [8], [9]. Didactic games, puzzles, and constructive materials are widely emphasized as learning environments that increase the complexity of children's cognitive actions—progressing from using ready-made models toward independently constructing new models and schemes [10], [12], [13]. In international early mathematics education, the learning-trajectories approach further strengthens this view by showing that conceptual growth is supported by carefully sequenced tasks and representations that match children's developmental pathways [14]. At the level of normative and methodological principles, standards-based frameworks also stress conceptual understanding, reasoning, representation, and connections—goals that align with modeling and problem-solving approaches [15]. Similarly, the realistic mathematics education tradition argues for grounding mathematics in meaningful activity and representations that help learners build conceptual structures from experience [16]. Reviews focused on early childhood mathematics education emphasize that effective programs combine play-based contexts with intentional, concept-rich activity and guidance [17].

The literature also supports an **integrative view** of preschool cognition that links sensory, intellectual, and creative development in a single developmental trajectory. In this view, sensory standards—identification, correlation, and perceptual modeling—provide a necessary base for stable perception and the discrimination of properties, enabling children to operate with forms, quantities, and relations in a systematic way [8], [12]. Intellectual actions develop through substitution (from objects to signs), the purposeful use of given models, and the ability to construct models as tools for solving tasks—an idea consistent with the stepwise formation of mental actions and the move toward internal regulation [6], [8]. Creative activity is interpreted not as "free fantasy" but as a productive component of cognition that supports generating alternatives, symbolizing relations, and producing original solutions—especially in puzzle-based, constructive, and combinatorial tasks [10], [13]. Within problem-centered pedagogies, TRIZ contributes an additional layer by formalizing search strategies and encouraging transformation of conditions and resources when facing difficulty, which can be pedagogically adapted to heuristic learning formats for children [11]. Taken together, these approaches provide a robust basis for designing mathematically meaningful play in which perception, reasoning, and creativity develop simultaneously through modeling, constructive activity, and guided exploration [8], [10], [12]–[14], [17].

6. RESEARCH METHODOLOGY

Methodological foundations

The study is grounded in:

- an **activity-based approach** (development through purposeful action and interaction);
- **humanistic preschool pedagogy** (choice, variability, individual pace, play-based meaning-making);
- the principle of **continuity** between preschool and primary mathematics;
- **TRIZ and heuristic learning principles** (problem situations, search strategies, transformation of means);
- **model-based learning** as a tool for revealing relations such as "part–whole," "form–transformation," and "quantity–comparison."

Methods

- **Theoretical methods:** analysis and synthesis of scientific sources; comparative analysis of preschool and early primary programs; conceptual modeling.
- **Empirical methods:** systematic observation of children's actions in modeling tasks and didactic games; analysis of children's products; elements of trial teaching and formative tasks.
- **Diagnostic tools:** tasks assessing recognition of shapes, composition analysis, symmetry, pattern construction, comparison of areas by partitioning, construction of nets for simple 3D compositions, and multiple-solution assembly tasks.

DISCUSSION OF RESEARCH RESULTS

1. Why modeling + TRIZ + heuristics is effective in early mathematics

The integration of TRIZ, heuristic learning, and mathematical modeling promotes a shift from reproductive learning toward exploratory cognition. Preschoolers develop not only "what" to know (names of shapes, numbers) but also "how" to think: how to detect patterns, generate alternatives, test solutions, and evaluate outcomes. Such activity makes intellectual effort natural and emotionally positive when adults cultivate curiosity, intrinsic motivation, and independence in searching for solutions.

2. The essence of mathematical modeling with preschoolers

In the preschool context, **mathematical modeling** can be defined as a teacher-organized, heuristically oriented process in which children create and transform models using simple planar and spatial abstractions. The model acts as a bridge between perception and concept: children learn to represent objects and relations, operate on representations, and gradually internalize these operations.

3. Classification of modeling techniques

Modeling practices can be categorized in two complementary ways:

A) By set-theoretical meaning

1. Constructing a whole of invariant form as a composition of different partition series (e.g., assembling a square from varied partitions).
2. Achieving discrete transformations of form through combinations of fixed partitions derived from an initial shape (e.g., tangram-like transformations).

B) By spatial orientation

1. **Planar modeling** (partitioning rectangles/squares).
2. **Spatial modeling** (partitioning rectangular parallelepipeds).
3. Modeling with **continuously deformable materials** (topological properties).
4. Modeling through **origami and flexagons** (folding transformations).

This classification provides teachers with a clear design map: from accessible, perceptual tasks to advanced transformation and construction tasks.

4. Developmental logic of cognitive abilities in modeling activity

Observation and pedagogical analysis show a stable developmental progression:

- **Younger preschool age:** immediate substitution and part–whole replacement using concrete objects and simple schemes.
- **Middle preschool age:** purposeful use of ready-made models; children learn to apply a given representation to solve a task.
- **Senior preschool age:** independent construction of model-schemes and invention of new models; children evaluate results against standards and regulate intermediate steps.

This progression connects to the growth of **logical–schematic thinking**—the ability to recognize relations and regularities and to operate with conditional algorithmic patterns (mental models).

5. Sensory, intellectual, and creative components (and why they must develop together)

In modeling tasks, sensory development includes:

- **identification** (matching perceived quality to a standard),
- **correlation** (matching when similarity is not obvious),
- **perceptual modeling** (recreating a quality using standard materials).

Intellectual development is supported by:

- **substitution** (objects → symbols),
- **use of given models** (adult-provided schemata),
- **construction of models** (child-built representations).

Creative development emerges through:

- **symbolization**,
- **detailing**,
- producing **original combinations** and alternative solutions.

The educational value lies in coordinated growth: children learn to see, represent, reason, and create within the same learning episode.

6. Diagnostic task types supporting preschool–primary continuity

The following task groups function simultaneously as learning activities and diagnostic indicators:

1. Decomposing complex figures into basic shapes; counting components (rectangles, triangles, circles, squares).
2. Constructing patterns/ornaments from geometric shapes.
3. Identifying symmetry relative to a given axis.
4. Building a figure symmetrical to a given one.
5. Comparing areas by partitioning shapes into equal polygons.
6. Creating nets of composite 3D figures (excluding cone/cylinder).
7. Analyzing equality, symmetry axes, and composition of figures.
8. Assembling a planar "box scheme" from a fixed set of shapes in multiple ways.

These tasks are consistent with early primary expectations and provide continuity without sacrificing preschool play-based forms.

6. The methodological backbone: "Set → Quantity → Number →
7. Relationship"

A key result is the justification of the sequence:
Set → Quantity → Number → Relationship
as a coherent progression for preschool mathematics content. It supports:

- introduction of quantity through actions with **sets**;
- development of comparison and measurement through **scalar quantities**;
- formation of **number** as a stable conceptual tool;
- understanding of **relations** (greater/less/equal, part–whole, "how many times," order, classification, symmetry).

This sequence strengthens conceptual clarity and reduces the risk of fragmented learning.

CONCLUSION

Integrating TRIZ, heuristic learning, and mathematical modeling offers a strong, modern strategy for preschool mathematics. It develops children's logical–schematic thinking, supports the formation of mathematical representations through active exploration, and builds self-regulation via comparison with standards and monitoring of outcomes. The proposed classification of modeling techniques and diagnostic task types can guide educators in designing age-appropriate, play-based, and individualized learning trajectories. The sequence **"Set → Quantity → Number → Relationship"** is presented as an effective methodological framework that ensures continuity between preschool and early primary mathematics while remaining compatible with humanistic and family-inclusive educational principles.

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