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## MODELING OF OSCILLATORY FLOW OF A RHEOLOGICALLY COMPLEX LIQUID IN A FLAT CHANNEL

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### Annotation

This research presents a detailed investigation into the laminar oscillatory flow of rheologically complex fluids within a flat channel. Such flows are characteristic of various industrial and technological processes involving polymer solutions, turbid water mixtures, and high-viscosity petroleum products. The study is grounded in the necessity to understand how the rheological properties of these fluids lead to unconventional hydrodynamic phenomena that differ from standard Newtonian behavior. The mathematical modeling is conducted using a simplified system of differential equations based on the generalized Maxwell model to account for the elastic-viscous properties of the medium. By employing periodic time functions, the research bypasses the need for initial conditions, focusing instead on stabilized oscillatory states. The study specifically analyzes the relationship between wall shear stress and the average velocity over the channel cross-section, deriving a transfer function known as the amplitude-spatial frequency characteristic (AFC). Numerical analysis reveals that at small oscillation frequencies, significant deviations in wall stress occur compared to Newtonian fluids. A key finding is the identification of a spatial phase shift in wall shear stress relative to the average velocity. Furthermore, the research demonstrates that under certain conditions—particularly with high relaxation times and specific fluid acceleration parameters—unusual effects such as reverse flow can occur within the channel. These results provide critical insights for the diagnosis and treatment of biological flows (such as blood) and the optimization of microfluidic devices.

### Abstract

This research presents a comprehensive analysis of the laminar oscillatory flow of rheologically complex fluids within a flat channel, a phenomenon prevalent in industrial and technological systems involving polymer solutions, turbid water mixtures, and high-viscosity petroleum products. The study is motivated by the fact that the inherent rheological properties of such fluids often trigger unconventional hydrodynamic effects that deviate significantly from classical Newtonian behavior.

The mathematical framework is constructed using a simplified system of differential equations based on the generalized Maxwell model to characterize the elastic-viscous nature of the medium. By utilizing periodic time functions, the model simplifies the complex system of equations, eliminating the necessity for initial conditions in stabilized flow scenarios. A key component of the analysis is the derivation of a transfer function, or amplitude-spatial frequency characteristic (AFC), which establishes the precise relationship between wall shear stress and the average flow velocity across the channel section. Numerical results demonstrate that at low oscillation frequencies, rheologically complex fluids exhibit substantial shifts in wall stress and spatial phase compared to Newtonian fluids. Furthermore, the study identifies that under specific conditions—characterized by high relaxation times (Deborah numbers) and varying fluid acceleration parameters—unique hydrodynamic phenomena, such as reverse flow, can occur within the channel. These findings offer significant implications for optimizing microfluidic devices, pneumatic micropumps, and biological diagnostic tools where precise flow control is paramount.

### Keywords

Rheological complex, oscillatory flow, differential equation, hydrodynamic effect, Maxwell model.



## 1. Introduction

The behavior of fluids flowing in channels and pipes used in technical and technological processes, including polymer liquids, turbid water mixtures, high-viscosity petroleum products, etc., forming an oscillating flow, depends on the rheological properties of the fluids, which in some cases can lead to unconventional hydrodynamic phenomena. Therefore, this research work addresses the specific problem of the oscillating flow of rheologically complex fluids in a flat channel. The main goal is to study the behavior of rheologically complex fluids on the basis of simplified mathematical models and compare the results obtained with the existing hydrodynamic laws in the oscillating flow of a Newtonian fluid in the special case, and as a result, to identify its different properties and to identify new hydrodynamic effects related to the hydrodynamic characteristics of rheologically complex fluids. It is known that oscillatory flows, in which transition processes do not occur in the flow of liquids, are of particular interest in science, in the fields of agriculture related to fluid motion, and in technological processes. Since the fluid motion in such processes is oscillatory, the process under consideration is a periodic function of time. In this case, it is assumed that the oscillations of the fluid occur in the same state at each period. Therefore, when solving problems related to the oscillatory flow of a liquid, periodic functions of time can be used, this property greatly simplifies the solution of the system of differential equations describing the oscillatory flow of rheologically complex fluids, and in such cases, it is not required to set initial conditions to solve the differential equation.

In recent years, biomechanical studies on the laminar oscillatory flow of blood, considered as a viscous fluid, have become an important factor in the diagnosis and treatment of the mechanism of human functioning and the targeted delivery of drugs to the vessels [1]. These studies are especially important for the effective use of biological microchips and pneumatic micropump devices used in the medical field [2]. In these devices, the fluid flow is considered as a laminar oscillatory flow of fluid in prismatic channels with a rectangular cross section. Unfortunately, there are not enough scientific research works on such flow issues. What exists consists of experimental results on the pulsating flow of fluid (blood) in microchannels. The scientific research works of Ye.P. Valieva, M.S. Purdin [3, 4] considered the issues of laminar pulsating flow and heat transfer in pipes and flat channels. Here, the main vibration-generating quantity is taken as a sinusoidal periodic time-varying function of the longitudinal velocity averaged over the channel cross section at the initial section. The problem is solved using the finite difference method. The solution results analyze the variation of the hydraulic resistance and wall shear stress in terms of vibration amplitude and phase. The analysis results are compared with the analytical solution in [5] and the results obtained through experiments, and it is found that the results are in good agreement.

Based on the Maxwell model, in [6-16], the problems of unsteady oscillatory flow of an elastic viscous fluid in a flat channel were considered. The problems were solved analytically. Calculation formulas for dynamic and frequency changes of quantities were derived. Using the determined formulas, the change of the wall shear stress depending on the vibration frequency parameter and the relaxation properties of the fluid was analyzed. Based on the analysis results, it was shown that taking the wall shear stress in the flow of a Newtonian fluid instead of the wall shear stress in the unsteady flow of the fluid leads to some limitations.

Despite the fact that the oscillatory flows in channels and pipes are considered as Newtonian fluids, very few research works have been devoted to the flows of rheologically complex fluids in this area. In this area, research works carried out by applying generalized Maxwell models to the oscillatory flow of rheologically complex fluids are insufficient. It is known that generalized Maxwell models play an important role in characterizing the oscillatory behavior of polymer solutions dissolved in water, turbid water mixtures, petroleum products, and other similar fluids.



Therefore, in this work, the problems of oscillatory flows of rheologically complex fluids in flat channels are studied based on generalized Maxwell models.

## 2. Problem formulation and solution methodology

Consider a flow in a flat channel where the distance between the walls is defined as  $2h$ , channel length  $L$  is defined as. Here  $L$  large enough, so that  $h/L \approx 1$ , the condition is met. In such cases, the flow is stabilized and the transverse velocity value is zero. The channel axes are defined as follows,  $X$  The axis is directed horizontally along the middle of the channel and is called the longitudinal axis,  $y$  and the arrow  $X$  is taken in a vertical direction perpendicular to the axis and is called the vertical axis.

Based on the above assumptions, the system of differential equations of oscillatory motion of a rheologically complex fluid, in one-dimensional space, is expressed in the form of a simplified system of differential equations of motion as follows [17-23].

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y}, \quad \frac{\partial p}{\partial y} = 0, \quad \tau = \alpha_1 \tau_1^0 + \alpha_2 \tau_2^0, \quad (1)$$

$$\tau_1^0 = \eta_1^0 \frac{\partial u}{\partial y}, \quad \lambda \frac{\partial \tau_2^0}{\partial t} + \tau_2^0 = \eta_2^0 \frac{\partial u}{\partial y}, \quad \eta = \alpha_1 \eta_1^0 + \alpha_2 \eta_2^0$$

Here  $u$  - longitudinal velocity;  $p$  - pressure;  $\rho$  - density;  $\tau$  - Stress;  $\alpha_1, \alpha_2$  - Concentrations of Newtonian and Maxwellian fluids;  $\tau_1^0, \tau_2^0$  - Real stresses of Newtonian and Maxwellian fluids;  $\eta_1^0, \eta_2^0$  - Newtonian and Maxwellian fluids have true dynamic viscosity coefficients;  $\eta$  - dynamic viscosity coefficient of the mixture;  $\lambda$  - relaxation coefficient;  $t$  - time variable.

Since the flow is symmetric about the longitudinal axis of the flat channel, the boundary conditions are formulated as follows:

$$y = h \text{ да } u = 0, \quad y = 0 \text{ да } \frac{\partial u}{\partial y} = 0. \quad (2)$$

In this problem, since a stabilized oscillatory flow of a fluid is considered, no initial conditions are required. The oscillatory motion of an elastic viscous fluid is caused by harmonic oscillations of a given fluid flow rate or velocity averaged over the channel cross section. That is:

$$Q = a_Q \cos \omega t = real a_Q e^{i \omega t}, \quad \langle u \rangle = a_u \cos \omega t = real a_u e^{i \omega t} \quad (3)$$

Here  $a_Q$  and  $a_u$  - the amplitude of the oscillation of the velocity averaged over the fluid flow rate and the channel cross section. Since the system of equations (1) is linear, the pressure, velocity and resultant stresses also oscillate in accordance with the fluid flow rate. Therefore, they can also be expressed as a periodic function of time in the following form

$$u(y, t) = real u_1(y) e^{i \omega t}, \quad p(x, t) = real p_1(x) e^{i \omega t}, \quad \tau(t) = real \tau_1 e^{i \omega t} \quad (4)$$

We substitute the expressions of the unknowns in the form (4) into the system of equations (1) and reduce it to this form:

$$\begin{aligned} i \omega u_1 &= -\frac{1}{\rho} \frac{\partial p_1}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_1}{\partial y}, \quad \tau_1 = \alpha_1 \tau_1^0 + \alpha_2 \tau_2^0, \quad \tau_1^0 = 2 \eta_1^0 D_{21}, \\ i \lambda \omega \tau_2^0 + \tau_2^0 &= 2 \eta_2^0 D_{21}, \quad D_{21} = \frac{1}{2} \frac{\partial u_1}{\partial y}, \quad \eta = \alpha_1 \eta_1^0 + \alpha_2 \eta_2^0 \end{aligned} \quad (5)$$

Using the second to fifth equations in the resulting system of equations (5), we form an equation of the following form to determine the product Stress:

$$\tau_1(y) = \eta \left( \frac{\alpha_1 \eta_1^0}{\eta} + \frac{\alpha_2 \eta_2^0}{\eta} \frac{1}{1 + i \omega \lambda} \right) \frac{\partial u_1}{\partial y}. \quad (6)$$

Substituting the found value of the applied Stress (6) into the first equation of the system of



equations (4), we obtain this equation:

$$\frac{\partial^2 u_1}{\partial y^2} - \frac{\rho i\omega \eta^* (i\omega)}{\eta} u_1 = \frac{\eta^* (i\omega)}{\eta} \frac{\partial p_1}{\partial x}. \quad (7)$$

Here

$$\eta^* (i\omega) = \left( \frac{\alpha_1 \eta_1^0}{\eta} + \frac{\alpha_2 \eta_2^0}{\eta} \frac{1}{1 + i\omega \lambda} \right)^{-1} = (X + Z \frac{1}{1 + i\omega \lambda})^{-1} =$$

$$\frac{1 + i\alpha^2 D e}{1 + i\alpha^2 D e X}, \quad \eta^* (i\omega) = \frac{1 + i\alpha^2 D e}{1 + i\alpha^2 D e X}, \quad X = \frac{\alpha_1 \eta_1^0}{\eta}, \quad Z = \frac{\alpha_2 \eta_2^0}{\eta}$$

The fundamental solutions of the homogeneous part of equation (7) are determined

$$\cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h}) \quad \text{and} \quad \sin(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h})$$

consists of functions, and the general solution of the homogeneous part is found in the following form:

$$u_1(y) = C_1(x) \cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h}) + C_2(x) \sin(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h}). \quad (8)$$

Its solution is sought in this

Since the non-homogeneous part of the equation is unchanged, form:  $u_1^* = A$ , in this case  $\frac{\partial^2 u_1^*}{\partial y^2} = 0$  is the solution to the inhomogeneous part of equation

(7)  $u_1^* = \frac{1}{\rho i\omega} \left( -\frac{\partial p_1}{\partial x} \right)$  Taking these into account, the general solution to the problem is determined as follows:

$$u_1(y) = C_1(x) \cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h}) + C_2(x) \sin(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h}) + \frac{1}{\rho i\omega} \left( -\frac{\partial p_1}{\partial x} \right) \quad (9)$$

The unknown integral coefficients in the solution (9) are found from the boundary condition (2). Taking into account the found coefficients, we determine the solution to equation (7) in the following form

$$u_1(y) = \frac{1}{\rho i\omega} \left( -\frac{\partial p_1}{\partial x} \right) 1 - \frac{\cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h})}{\cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))} \quad (10)$$

Here

$$\alpha_0 = \sqrt{\frac{\omega}{v}} h, \quad v = \frac{\eta}{\rho}.$$

For the solution found in (10), the final solution is determined as follows:

$$u_1(y, t) = \left( -\frac{\partial p_1}{\partial x} \right) \text{real} \frac{1}{\rho i\omega} 1 - \frac{\cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega) \frac{y}{h})}{\cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))} e^{i\omega t} \quad (11)$$

The solution (11) for the velocity is obtained by varying both sides of the solution.  $-h$  from  $+h$  We take the integral up to and derive the following formula to determine the Flow rate:

$$Q_1 = 2h \left( \frac{1}{\rho i\omega} \left( -\frac{\partial p_1}{\partial x} \right) \text{real} 1 - \frac{\sin(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))}{(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega)) \cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))} \right) e^{i\omega t} \quad (12)$$

$Q_1 = 2h < u_1 >$ , Taking into account that, from formula (12), we obtain a formula for determining the average velocity over the channel section

$$< u_1 > = \frac{1}{\rho i\omega} \left( -\frac{\partial p_1}{\partial x} \right) \text{real} 1 - \frac{\sin(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))}{(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega)) \cos(i^{\frac{3}{2}} \alpha_0 \eta^* (i\omega))} e^{i\omega t} \quad (13)$$

We find the resultant stress on the channel wall from the following equation

$$\tau_1 = -\frac{\eta}{\eta^* (i\omega)} \left. \frac{\partial u_1(y)}{\partial y} \right|_{y=h} \quad (14)$$



That is:

$$\tau_1 = -\frac{\partial p(x)}{\partial x} \operatorname{real}\left(\frac{\mu}{\rho i \omega h} \frac{i^{3/2} \alpha_0 \sin(i^{3/2} \alpha_0 \eta^*(i\omega))}{\eta^*(i\omega) \cos(i^{3/2} \alpha_0 \eta^*(i\omega))} e^{i\omega t}\right) \quad (15)$$

Here  $\rho i \omega = i \frac{\omega}{\nu} h^2 \frac{\mu}{h^2} = i \alpha_0^2 \frac{\mu}{h^2}$  Taking into account that, by eliminating the pressure gradient from formulas (13) and (15), we obtain a formula relating the resultant stress on the channel wall to the averaged velocity over the channel cross section

$$\langle u_1 \rangle = -\frac{h}{3\mu} \tau_1 \frac{3(i^2 \alpha_0 \eta^*(i\omega) \cos(i^2 \alpha_0 \eta^*(i\omega)) - \sin(i^2 \alpha_0 \eta^*(i\omega)))}{(i^2 \alpha_0)^2 \sin(i^2 \alpha_0 \eta^*(i\omega))} \quad (16)$$

From the defined formula (16) and  $W_{\tau_1, u_1}(i\omega) = \frac{\tau_1(i\omega)}{\langle u_1(i\omega) \rangle}$  function of passing from equality  $W_{\tau_1, u_1}(i\omega)$  We

derive the following formula for

$$W_{\tau_1, u_1}(i\omega) = \frac{h}{3\mu} \frac{\tau_1(i\omega)}{\langle u_1(i\omega) \rangle} = -\frac{(i^2 \alpha_0)^2 \sin(i^2 \alpha_0 \eta^*(i\omega))}{3(i^2 \alpha_0 \eta^*(i\omega) \cos(i^2 \alpha_0 \eta^*(i\omega)) - \sin(i^2 \alpha_0 \eta^*(i\omega)))} \quad (17)$$

The derived Transfer function in the form (17) is often called the amplitude-spatial frequency characteristic (AFC). The Transfer function determines the relationship between the wall stress and the averaged velocity over the channel section. In most cases, the fluid is in an unsteady flow, and to simplify the problem, the wall stress determined in a stationary flow is taken instead of the wall stress in the unsteady flow. This case can be true only for the case when the longitudinal velocity distribution is parabolic. In such a case  $\tau_{o, kc} = \frac{3\mu}{h} \langle u_1 \rangle$  is,  $\tau_{o, kc} = \tau_{hc}$  It can be taken as.

However, in other cases  $\tau_{o, kc} = \tau_{hc}$  can lead to significant errors or limitations. Therefore, in this research work, the case where the wall stress in an unsteady flow is different from the wall stress determined in a steady flow is investigated. We do this using the given transfer function formula (17).

### 3. Numerical calculation results and analysis

Using the transfer function formula (17), which was determined as a result of solving the problem of oscillatory flow of a rheologically complex fluid in a flat channel, we determine the relationship between the wall stress and the average velocity over the channel cross section in an unsteady flow. For this, we assume that the sinusoidal law of change of the average velocity over the channel cross section with respect to time is given. That is:

$$\langle u_1 \rangle = a_{u_1} \cos \omega t \quad (18)$$

Here  $a_{u_1}$  - the amplitude of the velocity averaged over the channel section. Using the given equation (18), we determine the relationship between the wall shear stress and the velocity averaged over the channel section. Since the given equation (18) is linear, the wall shear stress also oscillates in harmonic oscillation, but it is spatially shifted relative to the velocity averaged over the channel section. Therefore, we express the wall shear stress in this form

$$\tau_1 = a_{\tau_1} \cos(\omega t + \varphi_{\tau_1}) \quad (19)$$

Here  $a_{\tau_1}$  - wall specimen Stress amplitude;  $\varphi_{\tau_1}$  -  $\tau_1$  and  $\langle u_1 \rangle$  displacement space between sizes.

$$\cos(\omega t + \varphi_{\tau_1}) = \cos \omega t \cos \varphi_{\tau_1} - \sin \omega t \sin \varphi_{\tau_1}$$

Using the relationship and

$$\frac{\partial \langle u_1 \rangle}{\partial t} = -a_{u_1} \omega \sin \omega t$$

Taking into account that, we can reduce equation (19) to the following form

$$\tau_1 = \left( \frac{a_{\tau_1}}{a_{u_1}} \cos \varphi_{\tau_1} \right) \langle u_1 \rangle + \left( \frac{a_{\tau_1}}{a_{u_1}} \sin \varphi_{\tau_1} \right) \frac{1}{\omega} \frac{\partial \langle u_1 \rangle}{\partial t} \quad (20)$$



Here  $\left(\frac{a_{\tau_1}}{a_{u_1}} \cos \varphi_{\tau_1}\right)$  and  $\left(\frac{a_{\tau_1}}{a_{u_1}} \sin \varphi_{\tau_1}\right)$  The quantities (16) are the real and abstract parts of the Transfer function, respectively. Therefore, using equation (16), we obtain this equation

$$W_{\tau_1, u_1} = -\frac{1}{3} \left( \frac{-i\alpha_0^2 \sin(i^{3/2} \alpha_0 (\frac{1+iDe\alpha_0^2}{1+iDe\alpha_0^2 X})^{1/2})}{i^{3/2} \alpha_0 (\frac{1+iDe\alpha_0^2}{1+iDe\alpha_0^2 X})^{1/2} \cos(i^{3/2} \alpha_0 (\frac{1+iDe\alpha_0^2}{1+iDe\alpha_0^2 X})^{1/2}) - \sin(i^{3/2} \alpha_0 (\frac{1+iDe\alpha_0^2}{1+iDe\alpha_0^2 X})^{1/2})} \right) = \chi + \beta i \quad (21)$$

Here  $De = \frac{v\lambda}{h^2}$  - Deborah number, which represents the elasticity of a fluid;  $\chi, \beta$  is determined using these quantities, respectively:  $\chi = \left(\frac{a_{\tau_1}}{a_{u_1}} \cos \varphi_{\tau_1}\right)$ ,  $\beta = \left(\frac{a_{\tau_1}}{a_{u_1}} \sin \varphi_{\tau_1}\right)$

Taking these quantities into account, equation (20) becomes:

$$\frac{h}{3\mu} \frac{\tau_1}{\langle u_1 \rangle} = W_{\tau_1, u_1} = \chi + \beta \frac{1}{\omega} K_n \quad (22)$$

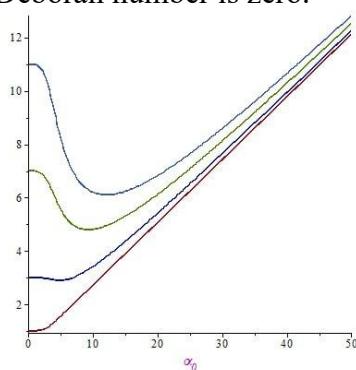
Here  $K_n = \frac{\partial \langle u_1 \rangle}{\partial t}$  - parameter characterizes the fluid acceleration,  $\chi$  I  $\beta$  - the quantities are determined from formula (21),  $t$  size  $t = \frac{h^2 \rho}{3\mu} t^*$  is reduced to dimensionless form by substitution.

As a result, equation (22) takes this form

$$\frac{\tau_{hc}}{\tau_{0kc}} = \chi + \frac{3\beta}{\alpha_0^2} K_n \quad (23)$$

Here  $\tau_{0kc} = \frac{3\mu}{h} \langle u_1 \rangle$  I  $\tau_1 = \tau_{hc}$  It was taken.

Based on the derived formula (23), Figure 1 shows the graphs of the change in the ratio of the wall stress in the unsteady flow to the wall stress in the steady flow depending on the oscillation frequency parameter for different values of the acceleration parameter and when the Deborah number is zero.



### 1. Picture

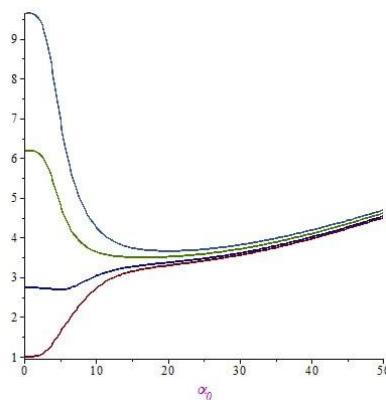
The ratio of the wall stress in a non-stationary flow to the wall stress in a stationary flow depends on the oscillation frequency parameter (for different values of the acceleration parameter and when the Deborah number is zero).

As can be seen from the graphs in Figure 1,  $K_n = 0$  when,  $\alpha_0$  at small values of  $\alpha_0$  the ratio  $\frac{\tau_{hc}}{\tau_{0kc}}$



changes by an amount close to one. If  $\alpha_0$  If it takes on values greater than one, then  $K_u = 0$  even though  $\frac{\tau_{hc}}{\tau_{0kc}}$  The ratio is greater than one and monotonically increases with increasing values of the oscillation frequency parameter. This shows that even at a value of the acceleration parameter equal to zero, the wall stress in the unsteady flow exceeds the wall stress in the steady flow.  $K_u$  with increasing parameter value  $\frac{\tau_{hc}}{\tau_{0kc}}$  The magnitude also increases, because in this case the wall shear stress is shifted forward in space relative to the velocity averaged over the channel cross section.

It is observed that the change in the shear stress in the unsteady flow of an elastic viscous fluid in a flat channel as a function of the Deborah number occurs at small values of the oscillation frequency parameter, with significant changes compared to the shear stress in the unsteady flow of a Newtonian fluid.



## 2. Picture

The ratio of the wall stress in a non-stationary flow to the wall stress in a stationary flow depends on the oscillation frequency parameter ( for different values of the acceleration parameter and the Deborah number  $De = 0.01$  and  $X = 0.1$  when).

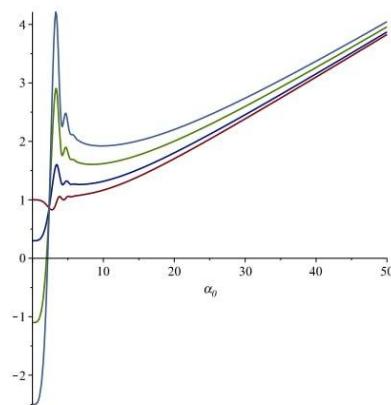
These changes are shown in Figures 2; 3; 4, based on equation (23), in which the ratio of the channel wall stress in the unsteady flow of an elastic viscous fluid in a flat channel to the channel wall stress in the steady flow of the fluid in this channel depends on the vibration frequency parameter.  $X = 0.1$  and at different values of the Deborah number. It can be seen from the graphs in Figure 2 that, unlike the Newtonian fluid flow, here, at values close to zero of the oscillation frequency parameter, an increase in the stress on the channel wall is observed depending on the fluid acceleration. For example, the fluid acceleration parameter  $K_u = 50$  in value  $\frac{\tau_{hc}}{\tau_{0kc}} = 9.5$  to, the parameter  $K_u = 30$  in value  $\frac{\tau_{hc}}{\tau_{0kc}} = 6.2$  will be equal. This size  $K_u$  At values of 0 and 10, it is equal to 1 and 2.7, respectively. At further increasing values of the vibration frequency parameter, the parameter  $K_u$  At values of 50 and 30  $\frac{\tau_{hc}}{\tau_{0kc}}$  decrease in size  $0 < \alpha_0 < 10.5$  It will be in the interval. After that  $\alpha_0$  as the parameter increases  $K_u$  at both values of  $\frac{\tau_{hc}}{\tau_{0kc}}$  The magnitude asymptotically approaches 4.  $K_u$  and at values of 0 and 10, the



vibration frequency parameter  $0 < \alpha_0 < 10.5$  in the intermediate values  $\frac{\tau_{hc}}{\tau_{0kc}}$  grows monotonically,

and at subsequent values of the parameter, this quantity asymptotically increases and approaches the number 4.

Deborah's number in Figure 3  $De = 0.1$  and  $X = 0.1$ . The change in the ratio of the channel wall stress in an unsteady flow to the channel wall stress in a steady flow is depicted as a function of the oscillation frequency parameter. The graphs in the figure show that at values of the oscillation frequency parameter close to zero, a sharp decrease in the channel wall stress is observed depending on the fluid acceleration.



### 3. Picture

The ratio of the channel wall stress in an unsteady flow to the channel wall stress in a steady flow depends on the oscillation frequency parameter (for different values of the acceleration parameter and the Deborah number  $De = 0.1$  and  $X = 0.1$  when).

In this case, the fluid acceleration parameter  $K_h = 50$  in value  $\frac{\tau_{hc}}{\tau_{0kc}}$  size  $\frac{\tau_{hc}}{\tau_{0kc}} = -2.5$  to, the

parameter  $K_h = 30$  in value  $\frac{\tau_{hc}}{\tau_{0kc}} = -1.2$  decreases to a negative value equal to . In this case, the flow

in the channel moves in the opposite direction.  $K_h$  At values of 0 and 10, there is no movement against the flow. Further increase in the oscillation frequency parameter  $2.5 < \alpha_0 < 5$  in the range of values, the parameter  $K_h$  When it's  $50 \frac{\tau_{hc}}{\tau_{0kc}}$  A sharp increase in magnitude occurs up to

the number 4, and then, its graph fluctuates and decreases. A similar situation occurs  $K_h$  This is also observed at 10 and 30, with the maximum increase reaching 1.6 and 2.8, respectively. After that  $\alpha_0$  as the parameter increases  $K_h$  at all values of  $\frac{\tau_{hc}}{\tau_{0kc}}$  The magnitude increases

monotonically. From the above analysis, it can be concluded that the main changes occur at large values of the relaxation time, due to the values of the oscillation frequency parameter close to zero. In such cases, even reverse flow can occur in the channel. This leads to the occurrence of unusual phenomena in the flow.

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