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APPLICATION OF TYPES OF CONVERGENCE OF RANDOM VARIABLES IN STATISTICAL ESTIMATION THEORY

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Abstract

This paper studies various types of convergence of sequences of random variables and their applications in mathematical statistics, particularly in the theory of statistical estimation. The concepts of convergence in probability, mean square convergence, and convergence in distribution are analyzed in relation to the consistency of estimators, the law of large numbers, and the central limit theorem. The practical importance of convergence theory in statistical methods is also discussed.

Keywords

random variable, convergence, statistical estimation, convergence in probability, convergence in distribution, law of large numbers, central limit theorem.

Introduction

In mathematical statistics, the study of the limiting properties of sequences of random variables is of great importance. In statistical research, it is often necessary to work with large samples. Under such conditions, determining the limiting properties of sample statistics helps assess the reliability of statistical estimations.

The types of convergence of random variables form the theoretical foundation of statistical methods. In particular, convergence in probability and convergence in distribution play an important role in studying the asymptotic properties of statistical estimators.

The aim of this paper is to analyze the application of different types of convergence of random variables in statistical estimation theory.

Main Part

In mathematical statistics, the study of limiting properties of sequences of random variables is essential. Statistical studies often require working with large samples, and in such cases, identifying the limiting behavior of sample statistics allows us to evaluate the reliability of estimators.

One of the central problems in mathematical statistics is estimation.

Suppose an observer has n observations $x_1,x_2,\ldots,x_n$ drawn from a population. These are considered independent and identically distributed random variables. The goal is to estimate an unknown parameter of the theoretical distribution such that the estimator provides an approximate value of this parameter. Any function of the sample is called a statistic or an empirical estimator of the unknown parameter.

For estimators to approximate the true parameter well, they must satisfy certain conditions.

Statistical estimators are of two types: point and interval estimators.

A **point estimator** is defined by a single numerical value. An **interval estimator** is defined by two values representing the bounds of an interval that contains the parameter.

Let $\theta^* = \theta^* (x_1,x_2,\ldots,x_n)$ be an estimator of the unknown parameter θ.

For large samples, estimators are required to be **consistent**. If, as $n\to\infty$, the estimator converges in probability to the true parameter, i.e., for any $\varepsilon>0$,

$$P\left(\left|\theta^* - \theta\right| > \varepsilon\right) \to 0,$$

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then θ^* is called a consistent estimator of θ. This also follows if the variance of θ^* tends to zero as $n \to \infty$.

Glivenko–Cantelli Theorem

The empirical distribution function $F_n(x)$ converges almost surely to the true distribution function $F(x)$:

$sup|F_n(x) - F(x)| \xrightarrow{a.s.} 0.$

This result provides the theoretical foundation for statistical inference based on empirical distributions.

Law of Large Numbers and Statistical Estimation

The law of large numbers is one of the key results in statistical estimation theory. It explains how the sample mean converges to the true expected value. In probability theory, the law of large numbers occupies a central place. It states that the sum or average of random variables follows a certain regular pattern in large samples. In statistical estimation, it forms the theoretical basis for parameter estimation.

If for a sequence $\{X_n\}$,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{p} E(X)$$

then the sequence satisfies the law of large numbers.

Theorem (Weak Law of Large Numbers):

If $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables with $E(X_i) = \mu$ and $Var(X_i) < \infty$, then

$\frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{p} \mu.$

Estimation of unknown parameters is one of the important issues in statistical estimation theory. The law of large numbers is important here in the following ways:

If $X_1, X_2, \ldots, X_n$ are random samples, then the sample mean

$\overline{X} = \frac{1}{n} X_i$

is taken as the estimate of the parameter $\mu = E(X_i)$. According to the law of large numbers:

$$\overline{X} \xrightarrow{p} \mu$$

This means that $\overline{X}$ is a reasonable estimate. Based on the law of large numbers, many statistical estimates are proved to be reasonable

Statistical significance:

- the sample mean estimates the parameter;
- statistical error decreases with larger samples;
- consistency of estimators is ensured.

Application of the Central Limit Theorem in Statistics

The central limit theorem is one of the fundamental results of probability theory and mathematical statistics. It states that the distribution of sums or averages of random variables approaches the normal distribution for large samples.

The standard normal distribution $N(0,1)$ has density:

$f(x) = \frac{1}{2\pi} e^{-\frac{x^2}{2}}.$

Theorem (Central Limit Theorem):

If $X_1, X_2, \ldots, X_n$ are i.i.d. with $E(X_i) = \mu$, $Var(X_i) < \infty$, then

$\frac{\sum_{i=1}^{n} X_i - n\mu}{\sigma\sqrt{n}} \xrightarrow{d} N(0,1).$

Applications in statistics:

The central limit theorem is widely used in statistics. It plays an important role in the following main areas:

The distribution of the sample mean:

If $\overline{X}=\frac{1}{n}\sum_{i=1}^{n} X_i$, then for large n

$\overline{X}\approx N(\mu,\frac{\sigma^2}{n})$.

It plays an important role in the following main areas:

- **Confidence intervals:**

The central limit theorem is used to construct confidence intervals for unknown parameters. For example, for μ:

$\overline{X}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$,

where $z_{\alpha/2}$ is the critical point of the standard normal distribution.

- **Hypothesis testing:**
- Test statistics often converge to normal distribution, ensuring reliable decision-making.
- **Practical applications:**
- economics, finance, insurance mathematics, regression analysis, and Big Data analysis.

Practical Importance of Convergence Theory in Statistical Methods

Convergence theory is widely applied in:

- parameter estimation;
- regression analysis;
- statistical testing;
- modeling economic and social processes;
- large-sample analysis.

It provides a theoretical foundation for the reliability and efficiency of statistical methods.

Applications in Financial Mathematics

Portfolio return modeling

Let a portfolio consist of n assets. The return on each asset is a random variable X_i, and the portfolio return is:

$R_n=\sum_{i=1}^{n} w_i X_i$,

where w_i is the share of the asset in the portfolio. Based on the central limit theorem for large n, R_n approaches a normal distribution, i.e

$R_n \xrightarrow{d} N(\mu,\sigma^2)$.

This allows for risk quantification for investors.

Option pricing and derivatives

Asset prices follow Geometric Brownian Motion:

$dS_t=\mu S_t dt+\sigma S_t dW_t$.

This leads to the Black–Scholes model.

Risk management.

Value at Risk (VaR):

$VaR_\alpha=inf\{x:P(L\leq x)\geq\alpha\}$,

where L is the potential loss, α is the confidence level. In large samples (L_n), the sequence approaches L.

Conclusion

The types of convergence of random variables form an essential theoretical foundation of mathematical statistics. Convergence in probability and convergence in distribution play a key role in studying asymptotic properties of estimators.

The law of large numbers justifies the convergence of the sample mean, while the central limit theorem explains the normal approximation of distributions. These results are widely used in hypothesis testing, confidence intervals, and statistical inference.

In financial mathematics, convergence theory is applied in portfolio analysis, option pricing, and risk management. It enables reliable modeling of stochastic financial processes and supports sound decision-making.

LIST OF REFERENCES USED

1. Sh.Q.Farmonov. “Ehtimollar nazariyasi”, Toshkent, “Universitet”, 2014.
2. Ross, Sheldon M. a first course in probability. Pearson Education, lnc.2010.
3. Robert B. Basic probability theory. Dover Publications, Inc.2008.
4. Боровков А.А.«Теория вероятностей», Москва, «Лань», 2010 г. Стр.705.
5. A.A.Abdushukurov, T.M.Zuparov. “Ehtimollar nazariyasi va matematik statistika”, Tafakkur bo‘stoni,2015 yil.
6. S.X. Sirojiddinov, M. Mamatov “Ehtimollar nazariyasi va matematik statistika”, Toshkent, “O‘qituvchi”, 1980 yil.
7. Севастянов Б.А. «Курс теории вероятностей и математической статистики», Москва, «Наука», 1982 г.