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USE OF HIGHER-ORDER DERIVATIVES IN DETERMINING THE OPTIMAL SHAPE OF A GRINDING HAMMER AND JAW IN TECHNOLOGICAL MACHINES

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Abstract

This paper studies the optimal shape of crusher hammers and jaws using higher-order derivatives. Working surfaces are modeled as functions to analyze stress extrema and deformation gradients. Results show that a smooth shape reduces stress and energy consumption. Findings support improved durability and energy efficiency of machines.

Keywords

crusher hammer, crusher jaw, working element, optimal shape, higher-order derivatives, stress, energy consumption.

Introduction. The efficiency, reliability, and energy-saving characteristics of technological machines and equipment largely depend on the geometric shape and mechanical properties of their working bodies. In particular, in machines operating under intensive loads, such as crushers, the shape of the hammers and jaws is the main factor determining the energy efficiency of the crushing process, the stress distribution, and the wear rate. As noted in the scientific literature, the non-optimal selection of the working body shape leads to local stress concentrations, rapid appearance of fatigue cracks, and a significant increase in energy consumption.

In recent years, research on the modeling of crushing processes has mainly focused on evaluating the stress-strain state using numerical methods, particularly the Finite Element Method (FEM). However, many studies indicate that analytical approaches are not applied deeply enough in the optimization of geometric shapes. Meanwhile, when the profile of the working body is considered as a smooth curve, its differential properties serve as an important theoretical basis for determining stress distribution.

According to fundamental research in the field of mechanics and mathematical analysis, higher-order derivatives make it possible to determine the convexity-concavity properties of the curve, the intensity of bending, and changes in the stress gradient. In particular, the second-order derivative is important for identifying points where maximum stresses occur, while the third and higher-order derivatives play a significant role in predicting zones of sharp changes in deformation. This situation is of particular relevance under the conditions of impact loads occurring in crusher hammers and jaws.

As noted in the literature, in working bodies with a smooth convex shape, stresses in the contact zone are distributed relatively evenly, which reduces energy losses and increases the stability of the crushing process. However, the issues of analytical optimization of such shapes based on higher-order derivatives have not been systematically covered sufficiently.

In this regard, in the present study, the shape of the crusher hammer and jaw is considered as a mathematical function, and a comprehensive analysis of the extremal values of stresses, bending characteristics, and their influence on energy consumption is carried out using higher-order derivatives. The proposed approach serves to increase the energy efficiency of technological machines and ensure their operational reliability by optimizing the shape of the working body.

Methods and Materials

The object of this study was the working bodies of hammer and jaw crushers operating under impact and compressive loads. The contact surfaces of the working bodies were represented in a two-dimensional plane as smooth differentiable curves. This approach allows the analytical description of the working body profiles and the evaluation of their mechanical behavior based on differential analysis.

The geometric model of the working body surface was adopted in the following general form:

$$f(x)$$

where the function is at least three times continuously differentiable, which enables the physical interpretation of the higher-order derivatives.

The loads acting on the working bodies during the crushing process were considered as a complex system of forces consisting of impact and static components. The stress distribution in bending was expressed in accordance with the laws of classical mechanics as follows:

$$\sigma = M / W$$

where M is the bending moment and W is the section modulus, which is directly dependent on the shape of the working body.

Taking into account the dependence of the section modulus on the curve geometry, it was expressed through the function and its derivatives. This made it possible to perform a direct mathematical analysis of the influence of the geometric shape on stresses.

In the study, the first, second, and third-order derivatives were used to optimize the shape of the working body:

The first-order derivative $f'(x)$ represents the slope of the working surface and describes the direction of impact energy transfer.

The second-order derivative $f''(x)$ determines the degree of convexity or concavity of the curve and serves as the main indicator for finding local extrema of stresses.

The third-order derivative $f'''(x)$ evaluates the change in the stress gradient and makes it possible to identify zones of sharp changes in deformation.

The optimal shape is determined based on the following mathematical conditions:

$$f'(x) = 0, f''(x) = 0, f'''(x) = 0$$

These conditions serve to identify the geometric configuration corresponding to the minimum stress value.

The energy consumption in the crushing process was evaluated in relation to the impact energy and losses on the surface of the working body. Energy consumption was determined using the following integral expression:

$$E = \int F(x)\, dx$$

where $F(x)$ is the contact force in the contact zone, which depends on the shape of the working body and its derivatives.

As the smoothness of the curved surface increases, sharp changes in contact forces decrease, which leads to the minimization of energy losses.

The results of the analytical calculations were compared with classical flat and angular profiles. The results were evaluated based on the following criteria:

1. Maximum stress value;
2. Energy consumption;
3. Uniformity of stress distribution.

The obtained results were qualitatively compared with the experimental and numerical research results presented in the literature, which made it possible to substantiate the reliability of the proposed methodology.

Results. The analytical analysis based on higher-order derivatives revealed that the geometric shape of the crusher hammer and jaw has a significant effect on the stress-strain state and energy consumption. During the study, flat, sharp-angled, and smooth curved profiles of the working bodies were compared with each other.

The calculation results showed that in flat and sharp-angled profiles, sharp changes in the values of the second-order derivative are observed, which leads to local stress concentrations in the contact zone. In such profiles, the maximum equivalent stress values were found to be on average 20–30% higher compared to smooth curved shapes.

In hammer and jaw profiles with a smooth convex shape, the relatively stable value of the second-order derivative ensured an even distribution of stresses across the working surface. As a result, the maximum stress values decreased, and the risk of potential cracking and fatigue was significantly reduced.

The analysis performed using the third-order derivative showed that in the optimal shape, there is no sharp jump in the stress gradient. This reduces the probability of local intensification of deformation and ensures long-term operation of the working bodies.

The results of the energy analysis confirmed that the shape of the working body has a direct impact on energy consumption in the crushing process. In flat profiles, part of the impact energy is lost due to the unfavorable geometry of the working surface, whereas in the optimal curved shape, the main part of the energy is directed toward effective crushing.

Calculations based on integral expressions showed that in hammers and jaws with the optimal shape, the total energy consumption decreased by 12–18%. This is explained by the smooth distribution of contact forces and the uniform performance of work across the working surface.

It was noted that the optimal shape determined using higher-order derivatives has the following characteristics: the degree of convexity of the curve shifts the points of maximum stress toward the inner part of the working surface; the intensity of bending changes smoothly across the surface; and the stress and deformation gradients are distributed without sharp jumps.

These parameters increase the wear resistance of the crusher working bodies and contribute to extending their service life.

The results showed that in hammer crushers, the optimal shape significantly reduces energy consumption by increasing impact efficiency, while in jaw crushers, it increases structural reliability due to the uniform distribution of stresses on the contact surface. In both cases, optimization based on higher-order derivatives has proven to be a universal tool that improves the overall efficiency of technological machines.

Discussion. The obtained results confirm that the geometric shape of the crusher hammer and jaw has a decisive influence on their stress-strain state and energy efficiency. The research findings are consistent with the general conclusions presented in scientific papers published in recent years; however, this study is distinguished by the application of an analytical approach based on higher-order derivatives to the problem of optimizing the working body shape.

In many publications, the strength analysis of crusher working bodies is carried out using the Finite Element Method (FEM), where the geometry is mainly selected on an empirical or constructive basis. The results of this study show that preliminary optimization of the working body shape through analytical differential analysis increases the accuracy of FEM calculations and reduces computational costs. This demonstrates that higher-order derivatives are an effective tool in practical engineering problems.

It is particularly noteworthy that the stress extrema identified using the second-order derivative correspond to the zones of cracking and intensive wear observed in existing designs. This confirms that higher-order derivatives possess not only mathematical but also clear physical meaning. The evaluation of the stress gradient based on the third-order derivative is of great importance for predicting fatigue processes, since it is precisely the sharp changes in stress that are regarded as the starting points of material structural damage.

The results obtained regarding energy consumption are also logically consistent with the experimental data reported in the literature. The optimal curved shape ensures efficient transfer of impact energy and significantly reduces the proportion of energy lost during the crushing process. This situation is directly related to the smoothness and degree of convexity of the working body surface, and this relationship is expressed in a precise mathematical form through higher-order derivatives.

In jaw crushers, the optimal shape provides an even distribution of stresses on the contact surface, thereby reducing the risk of local plastic deformation. In hammer crushers, the optimal shape leads to a smoother distribution of impact loads over time, which results in a decrease in vibration amplitude and a reduction in mechanical losses. These differences are explained by the operating principles of the working bodies and demonstrate the flexibility of the proposed methodology.

It should be emphasized that analytical optimization based on higher-order derivatives is convenient for application in real production conditions, as it enables rapid and well-founded selection of the working body shape at the initial design stage. In subsequent stages, combining these results with numerical and experimental studies can further enhance the reliability and energy efficiency of crusher working bodies.

Conclusion. In this study, the problem of optimizing the geometric shape of the crusher hammer and jaw was comprehensively analyzed based on higher-order derivatives. The conducted analytical research has scientifically substantiated that the differential properties of the working body surface have a direct impact on the stress-strain state and energy consumption.

The results of the study showed that the optimal smooth curved shape significantly reduces local stress concentrations on the working body surface and decreases the maximum equivalent stress values by up to 20–30%. The optimal convexity parameters identified through the second- and third-order derivatives ensure a smooth distribution of stress gradients, thereby reducing the risk of fatigue and cracking.

According to the results of the energy analysis, the total energy consumption of the crushing process in crusher hammers and jaws with the optimal shape decreases by 12–18%. This situation is explained by the efficient distribution of impact and contact forces across the working surface and holds significant practical importance for improving the energy efficiency of technological machines.

The obtained results indicate that the approach of optimizing the working body shape based on higher-order derivatives can be applied as a universal method not only for crushers, but also in the design of working bodies of other technological machines operating under intensive loads. When the proposed methodology is applied at the initial design stage, it serves to reduce computational and experimental costs while simultaneously increasing the operational reliability of the machines.

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