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ABOUT T-STATISTICS AND THE THEORY OF T-TESTS ON IT

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Abstract. This paper investigates the theory of the *t-statistic* and *t-tests*, which are widely used in mathematical statistics. The theoretical foundations of the t-test are justified using the **Central Limit Theorem (CLT)** and **Slutsky's Theorem**. The importance of the t-test in estimating population parameters and testing statistical hypotheses in small samples is demonstrated. In addition, one-sample, two-sample, and paired-sample t-tests are analyzed.

Introduction

One of the fundamental tasks of mathematical statistics is to estimate population parameters based on a sample and to test hypotheses about them. In practice, the population variance is often unknown and the sample size is small. For this reason, the *t-statistic*, introduced by Student, plays a crucial role. In this paper, the theoretical foundations of the t-test are studied in connection with the Central Limit Theorem and Slutsky's Theorem.

Basic Concepts

Definition of the t-statistic. Let $X_1, X_2,\ldots,X_n$ be random variables. The sample mean and sample variance are defined as

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and } S^2=\frac{1}{n-1}\sum_{i=1}^{n} (X_i-\overline{X})^2.$$

Assume that $X_1, X_2,\ldots,X_n$ are independent and identically distributed random variables with a normal distribution

$$X_i \sim N(\mu,\sigma^2).$$

If the standard deviation σ^2 is unknown, the following statistic is introduced:

$$t=\frac{\overline{X}-\mu}{S/\sqrt{n}}.$$

Theorem (Student). If $X_i \sim N(\mu,\sigma^2)$, then the t statistics above has a Student's distribution with $(n-1)$ degress of freedom.

Central Limit Theorem (CLT). If $X_1, X_2,\ldots,X_n$ are independent and identically distributed random variables with finite mean and variance, then

$$\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1).$$

This result justifies the normal approximation for large samples.

In probability theory and mathematical statistics, the concept of convergence of random variables plays an essential role. Slutsky's Theorem is one of the key results describing relationships between different types of convergence and is widely used in the justification of statistical methods.

Slutsky's Theorem. If

$$X_n \xrightarrow{d} X, \ \ Y_n \xrightarrow{p} c,$$

then

$$\frac{X_n}{Y_{n_n}} \xrightarrow{d} \frac{X}{c}.$$

Slutsky's Theorem is an important tool for deriving limiting distributions in statistics. It allows replacing unknown parameters with consistent estimators.

For many statistical estimators, the following convergence holds:

$$\sqrt{n}(\widehat{\theta}_n-\theta) \xrightarrow{d} N(0,\sigma^2).$$

If the variance is unknown and replaced by an estimator $\hat{\sigma}_n$, then by Slutsky's Theorem:
$\frac{\sqrt{n}(\hat{\theta}_n-\theta)}{\hat{\sigma}_n}\xrightarrow{d} N(0,1)$.

Using Slutsky's Theorem, the theoretical justification of Student's t-test, Wald test, and Z-test can be established.

In linear regression models, estimators are asymptotically normally distributed. When the variance is replaced by its estimator, statistical tests are justified via Slutsky's Theorem.

Theoretical Justification of the t-statistic

By the Central Limit Theorem,
$\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\approx N(0,1)$.

However, σ is unknown. Replacing it with s, we obtain:
$t=\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$.

By Slutsky's Theorem,
$t\xrightarrow{d} N(0,1)$.

For small samples,
$t\sim t_{n-1}$.

One-sample t-test

Hypothesis: $H_0:\mu=\mu_0$.

Test statistic: $t=\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$.

Decision rule: $|t|>t_{\frac{\alpha}{2},n-1}$.

Two-sample t-test

Let two independent samples be given:
$X_1, X_2,\ldots,X_n\sim N(\mu_1,\sigma^2),\quad Y_1,Y_2\ldots,Y_n\sim N(\mu_2,\sigma^2)$.

Hypothesis: $H_0:\mu_1=\mu_2$.

Test statistic: $t=\frac{\overline{X}-\overline{Y}}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}}$.

where the pooled variance is

$$S_p^2=\frac{(n-1)S_X^2+(n-1)S_Y^2}{n+m-2}.$$

Paired t-test

If observations are given in pairs, define:
$D_i=X_i-Y_i$.

Then
$t=\frac{\overline{D}}{S_D/\sqrt{n}}$,

where $\overline{D}=\frac{1}{n}\sum D_i$ is the variance of differences.

Confidence Interval
$\overline{X}\pm t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}$.

For large n:
$t\approx N(0,1)$.

This follows from the CLT and Slutsky's Theorem.

Applications. The t-test is widely used in economics and econometrics, medicine and biology, engineering experiments, and social sciences.

Results and Discussion. The t-test is an optimal tool for small samples and remains applicable for large samples based on the Central Limit Theorem. Slutsky's Theorem strengthens its theoretical foundation.

Conclusion. In this paper, the t-statistic has been theoretically justified. Its asymptotic properties were established using the Central Limit Theorem and Slutsky's Theorem. The t-statistic is an important tool for statistical analysis in small samples. It enables reliable estimation and hypothesis testing when the variance is unknown. Among modern statistical methods, the t-test stands out due to its simplicity and effectiveness.

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