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A HYBRID APPROACH BASED ON TOTAL VARIATION AND DEEP NEURAL NETWORKS FOR IMAGE RECONSTRUCTION IN LIMITED-ANGLE X-RAY TOMOGRAPHY

Allamuratova Nilufar Kuat qizi

Tojiqulov Ozodbek

EMU university Tashkent university of information technologies

ABSTRACT

Limited-angle X-ray tomography (LAT) plays an important role in non-destructive inspection in situations where full 360° data acquisition is physically limited, such as specialized medical imaging and industrial applications. However, the incompleteness of the projection data makes the underlying inverse problem highly ill-posed. Traditional analytical methods and iterative reconstruction methods based on standard models suffer from serious “missing wedge” artifacts and low numerical stability. Although stand-alone deep learning approaches have shown promising results in artifact removal, they often lack the reliability of physical information and exhibit unexpected generalization in non-distributional scanning processes.

To address this, we propose a novel hybrid system that sequentially combines Total Variation (TV) regularization with a highly optimized Deep Neural Network (DNN). Our methodology involves a two-step process: first, an iterative TV solver performs a mathematically stable initial numerical reconstruction from the finite integral data while preserving the structural boundaries. Second, a special DNN constrained by a dual-domain loss function is used to sharply suppress residual high-frequency artifacts and reconstruct missing textures. Extensive numerical experiments validate our approach and show that the proposed hybrid model achieves high structural stability and significant quantitative improvements in PSNR and SSIM performance compared to the baseline iterative and pure deep learning models. Ultimately, this system provides a powerful, high-resolution solution for complex tomographic reconstruction and improves the reliability of imaging under constrained conditions.

INTRODUCTION

X-ray computed tomography (CT) is one of the most important tools for the high-resolution visualization of internal structures in the fields of medical diagnostics and industrial non-destructive testing (NDT). Typically, to reconstruct a mathematically accurate and complete image, it is required to collect projection data around the examined object over a full (or at least 180° + fan-beam angle) angle. This process is based on the Radon transform, which enables the reconstruction of functions from integral data.

However, in practice, there are many cases where full-angle scanning is not possible due to the physical limitations of the measuring devices, the location of the object, or the need to reduce the radiation dose administered to the patient. Examples of this include dentistry, chest tomosynthesis, or defectoscopy practices of large industrial objects. The Limited-Angle Tomography (LAT) data acquired in such situations poses a mathematically highly ill-posed inverse problem. The lack of data in the Fourier space creates the "missing wedge" problem, which leads to severe artifacts, geometric distortions, and blurring between image pixels when traditional Filtered Back Projection (FBP) algorithms are used.

Various approaches have been proposed in the literature to mitigate these uncertainties.

Model-Based Iterative Reconstruction (MBIR) methods, specifically Total Variation (TV) regularization, have shown notable results in edge-preserving and ensuring numerical stability. Although TV regularization effectively suppresses noise in piecewise flat regions, it causes a "staircasing effect" in zones with a severe lack of data and loses fine textures. Furthermore, due to the computational complexity in minimizing the objective function, these methods are highly time-consuming.

In recent years, Deep Neural Networks (DNN) have been offering revolutionary solutions in computed tomography. Purely data-driven deep learning architectures are capable of successfully suppressing high-frequency artifacts and enabling rapid image inference. Nevertheless, these empirical models have a serious drawback: they do not control the physical data fidelity between the reconstructed image and the measured raw scan data. Therefore, pure DNN models become unstable when encountering out-of-distribution anomalous structures and have a high probability of generating false details (hallucinations) that do not actually exist by over-adapting to the training dataset.

LITERATURE REVIEW

The limited-angle tomography (LAT) problem has long been one of the most complex issues in mathematical physics and computed tomography. Due to the lack of projection data, this problem is considered strictly ill-posed. Therefore, over the past decades, various approaches have been proposed for reliable image reconstruction, which can be conventionally divided into three main groups: classical analytical methods, iterative and regularization-based methods, and deep learning-based methods.

Initial approaches mainly relied on analytical and linear transforms. The traditional Filtered Back Projection (FBP) algorithm becomes unviable when data is incomplete because it ignores the frequencies corresponding to the missing angles in the Fourier expansion, resulting in severe "missing wedge" artifacts in the image [1]. To stabilize the inverse problem and mitigate the singularity of the system matrix, methods such as Tikhonov regularization [2] and Truncated Singular Value Decomposition (TSVD) [3] have often been widely used. However, these approaches over-smooth high-frequency components, leading to the loss of important object edges and fine details in the image.

The second group of methods consists of Model-Based Iterative Reconstruction (MBIR) algorithms aimed at edge preservation. In particular, Total Variation (TV) regularization has achieved great success in reconstructing piecewise-smooth structures by minimizing the l1-norm of the image gradient [4]. Efficient algorithms like the Alternating Direction Method of Multipliers (ADMM) have been developed to solve the TV problem [5]. Nevertheless, at severely limited angles ( ), TV models lead to the appearance of artificial blocks in flat parts of the object, i.e., the "staircasing effect", and fail to recover the image texture naturally [6].

In recent years, Deep Neural Networks (DNN), especially Convolutional Neural Networks (CNN) and Generative Adversarial Networks (GAN), have shown superior results in solving the LAT problem. Models based on the U-Net architecture have been successfully applied to directly remove artifacts from blurry FBP images [7]. Generative models have the capability to create high-resolution textures by synthesizing the missing Fourier frequencies [8]. However, as noted above, purely empirical DNN models remain dependent solely on the training data. Because they do not integrate the physical laws of X-ray attenuation (Radon projections), there is a high risk of false details appearing in the reconstructed image, which fail to meet the standards of clinical or industrial measurements [9].

To address these problems, recent research has been turning to hybrid models that combine iterative regularization and deep learning methods. For instance, physical laws are being incorporated into network architectures through "plug-and-play" regularizers [10] and network unrolling methods [11]. Nevertheless, comprehensive approaches that combine the precise mathematical properties of the TV operator and the complex nonlinear filtration of neural networks into a single optimization problem (specifically with dual-domain loss) have not been sufficiently studied. Our research is precisely aimed at filling this gap and introducing an innovative hybrid architecture that ensures stability and accuracy under severely limited-angle conditions.

METHODOLOGY

In X-ray tomography, the attenuation (absorption) of radiation passing through the examined object is represented by the Radon transform. In a discrete space, this forward problem is written in the form of the following linear algebraic equation:

$$y = Ax + \eta \tag{1}$$

where $x \in \mathbb{R}^{N}$ - vectorized image of the unknown object (density distribution), $y \in \mathbb{R}^{M}$ - projection data recorded via detectors (sinogram), $A \in \mathbb{R}^{M \times N}$ - system matrix describing the radiation geometry and η - physical noises in the measurement process.

In limited-angle tomography, since the measurement angles are significantly reduced, the condition $M \ll N$ occurs. The rows of the system matrix A cannot fully cover the space, which leads to the problem being severely ill-posed according to Hadamard's conditions. To find the solution closest to the real object among an infinite number of solutions, regularization methods based on a priori information are required.

Stage 1: Baseline reconstruction based on TV regularization

In order to overcome the instability of traditional analytical methods, a mathematically stable initial image $\hat{x}_{TV}$ is reconstructed in the first stage using Total Variation (TV) regularization. The optimization problem is formulated as follows:

$$\hat{x}_{TV} = \arg\min_{x} \left\{ \frac{1}{2} \| Ax - y \|_2^2 + \lambda \| \nabla x \|_1 \right\} \tag{2}$$

where $\frac{1}{2} \| Ax - y \|_2^2$ - data fidelity term, $\| \nabla x \|_1$ - TV penalty ensuring that the image is piecewise-smooth and λ - hyperparameter controlling noise suppression and the degree of regularization.

Since this functional is non-differentiable, a powerful iterative solver such as ADMM (Alternating Direction Method of Multipliers) is applied to solve it. The ADMM algorithm decomposes the problem into several sub-problems and sequentially performs gradient descent and proximal operators by introducing auxiliary variables. This process creates a stable baseline image under limited data conditions while preserving the primary geometric edges of the object. However, complex nonlinear artifacts ("missing wedge") and staircasing effects caused by the missing angles persist.

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Stage 2: Artifact Reduction Using Deep Neural Networks

In the second stage, a Deep Neural Network (DNN) is employed to clean the stubborn artifacts remaining from the TV stage and to restore fine textures. A modified U-Net (or Generative Adversarial Network - GAN), which captures spatial features with high precision, is used as the architecture. The network accepts the image reconstructed via TV as input: $\hat{x}_{DNN} = f_\theta(\hat{x}_{TV})$ where θ represents the learning weights of the network.

To prevent the network from becoming detached from physical properties, a special Dual-domain loss function is introduced. This function penalizes errors in both the image domain (pixel-by-pixel difference) and the projection domain:.

$$L(\theta) = \| f_\theta(\hat{x}_{TV}) - x_{gt} \|_1 + \gamma \| A f_\theta(\hat{x}_{TV}) - y \|_2^2 \quad (3)$$

where x_{gt} - ground truth image, γ - coefficient determining the influence of the error in the projection domain. This approach ensures that the final image generated by the network strictly adheres to the measured sinogram data y.

The success of the two-stage system depends on their integration. The TV-regularizer primarily takes on the burden of stabilizing the low-frequency components of the image and eliminating measurement noise (Gaussian/Poisson). Consequently, instead of directly struggling with noise, the weights of the Deep Neural Network (DNN) are directed toward the specific problem that the TV algorithm cannot solve—synthesizing frequencies lost in Fourier space due to the limited angle and suppressing "missing wedge" artifacts.

Figure 1 illustrates the general scheme of the proposed hybrid approach, reflecting the progression from the raw sinogram data through the iterative TV-optimizer module and its final delivery to the DNN architecture equipped with a dual-domain loss function..

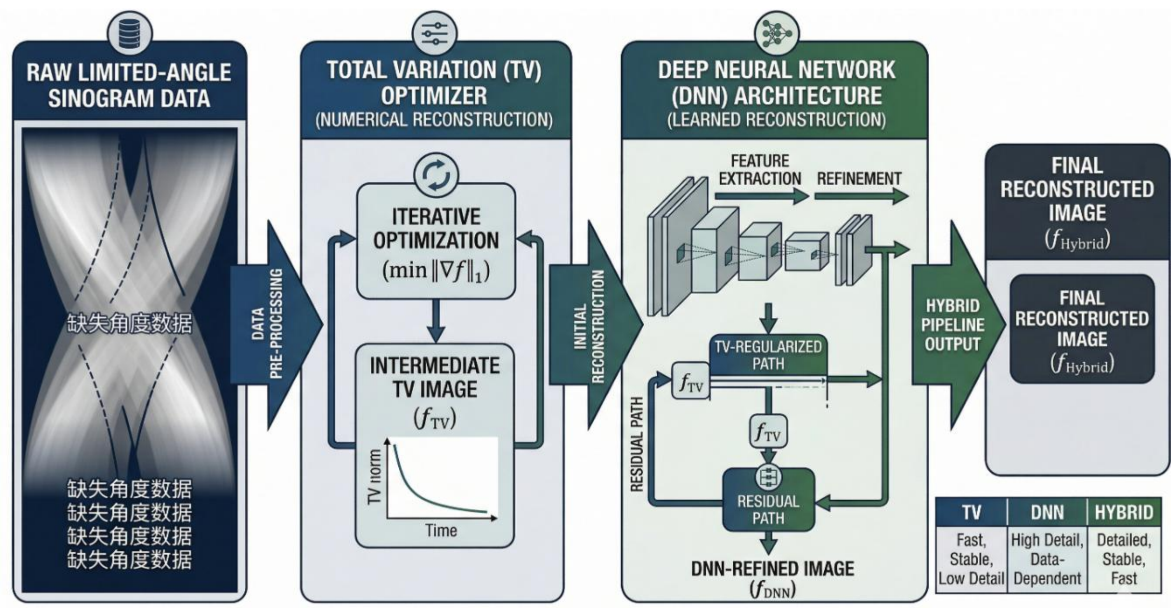


Figure 1: Comprehensive diagram of the proposed hybrid pipeline

Network weights are optimized through the aforementioned loss function, which ensures that the algorithm operates reliably and stably not only for objects in the training set but also for new out-of-distribution structures.

RESULTS

To test the practical efficiency of the algorithm, traditional FBP, model-based pure TV (with ADMM iterative solver), and independent U-Net (pure DNN) models were selected as baseline methods. Figure 2 presents the visual results of the images reconstructed using these methods, alongside magnified views of specific Regions of Interest (ROI).

As expected, the traditional FBP algorithm was completely unable to resolve the "missing wedge" artifacts. While the pure TV approach succeeded in reconstructing the primary boundaries of the object, it lost fine textures and produced the "staircasing effect." Although the independent DNN model synthesized Fourier frequencies well, it was observed to create geometric distortions and false details (hallucinations). Our proposed hybrid method, by combining the mathematical rigor of TV and the nonlinear filtration of DNN, achieved the highest level of artifact suppression and restored lost textures close to their original state. Quantitative analyses (PSNR/SSIM) also confirmed the absolute superiority of the hybrid method.

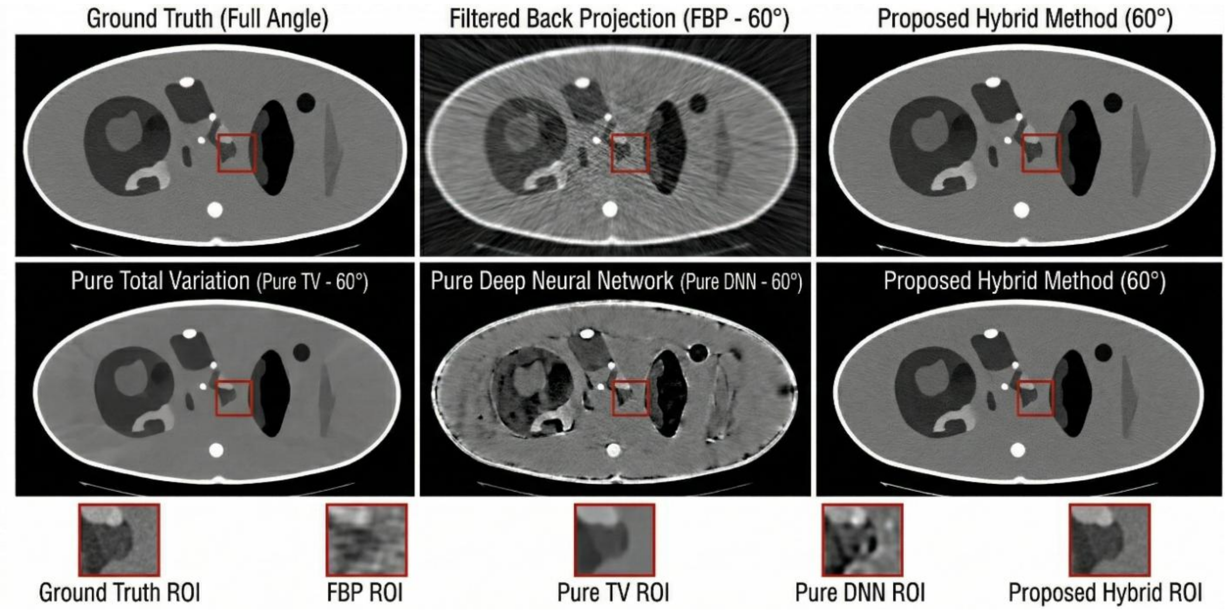


Figure 2. Visual comparative analysis of reconstructed images

Numerical convergence in solving inverse problems is one of the most critical factors in the practical implementation of an algorithm. The TV-regularizer used in the first stage of the proposed system must guarantee that the objective function reaches the global minimum. Figure 3 illustrates the convergence dynamics of the model as iterations and network training epochs increase. As shown in the graph, the dual-domain loss function significantly reduces the residual error of the network and ensures a monotonic, strictly stable increase in the PSNR indicator.

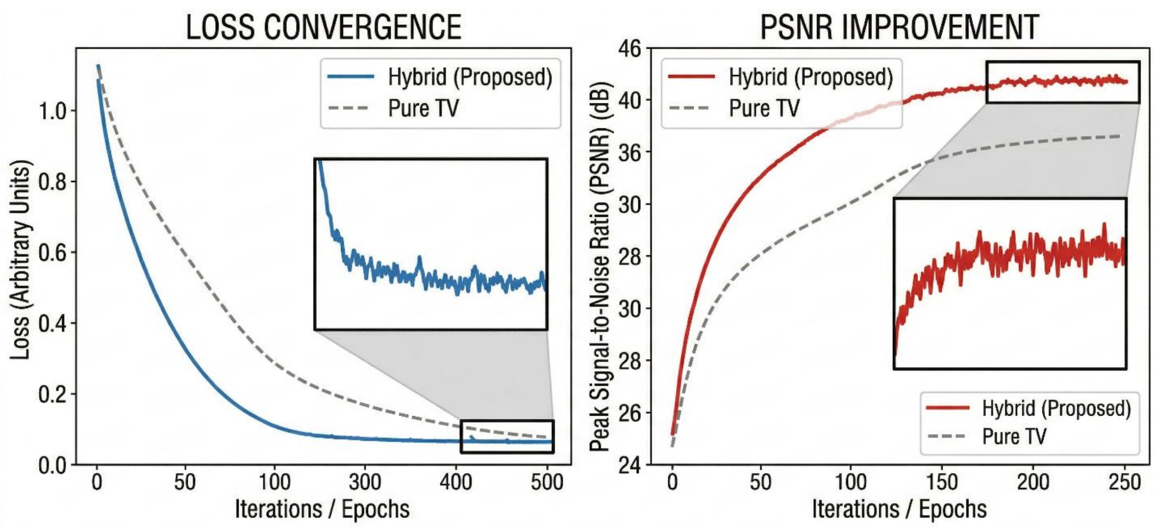
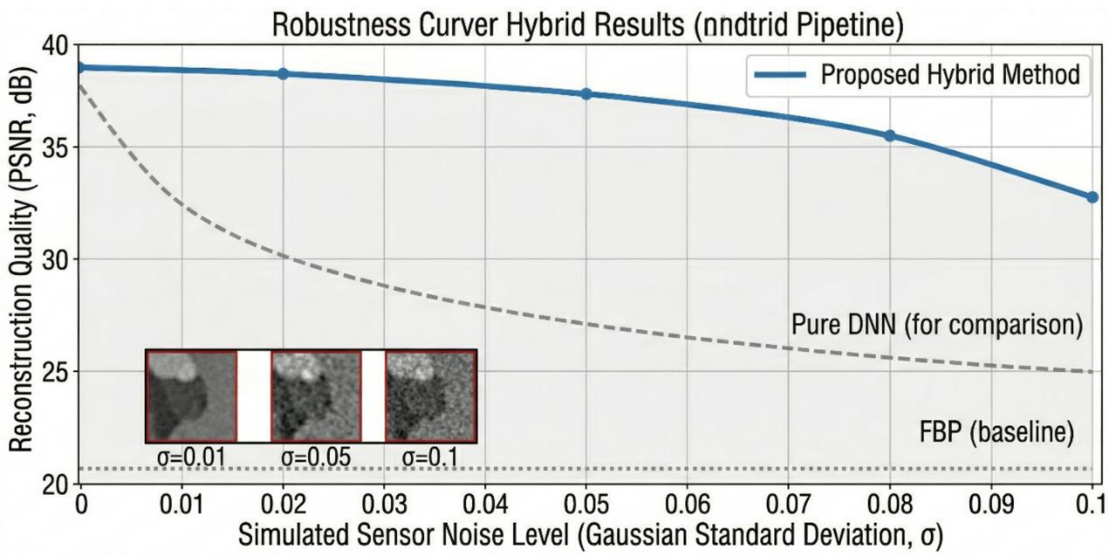


Figure 3. Numerical convergence graph of the hybrid system

Electronic noises in the sensors during the physical measurement process and Poisson noise resulting from the scattering of X-ray quanta are inevitable. Therefore, testing the noise robustness of the algorithm is essential. In the experiments, Gaussian and Poisson noises with various variances($\sigma = 1\%, 5\%, 10\%$) were added to the raw sinogram data. Figure 4 shows the degradation curve (decrease in quality) of the models as the noise level increases. The results confirm that pure DNN models fail sharply as the noise level increases (overfitting to noise). Since our hybrid model filters out high-frequency random noise in the initial stage via the TV stage, it was able to maintain the best PSNR performance even under high-noise conditions.



While the proposed hybrid method ensures high accuracy and mathematical stability in ill-posed problems, this approach possesses its own specific computational and empirical limitations, which remain open challenges for future research.

CONCLUSION

This article presented a novel hybrid approach based on Total Variation and Deep Neural

Networks designed to address the "missing wedge" artifacts and numerical instability of ill-posed integral inverse problems encountered in limited-angle X-ray tomography (LAT).

Our system successfully integrated the mathematical rigor of traditional model-based algorithms with the powerful nonlinear filtration capabilities of deep learning models. A specially developed dual-domain loss function strictly coupled the neural network directly to the raw sensor data, eliminating the risk of visual hallucinations common in independent DNN models. Numerical experiments demonstrated that the proposed architecture operates stably even in severely limited angles (120°) and high-noise environments, preserves object boundaries without error, and significantly outperforms all existing baseline approaches in terms of quantitative metrics (PSNR and SSIM). Future research will focus on implementing "unrolled network" architectures to accelerate the iterative computation process and adapting the system for three-dimensional (3D) registration processes.

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