JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 14 Issue 12, December 2024
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

LIMIT THEOREM FOR FLUCTUATIONS OF NEARLY CRITICAL BRANCHING PROCESSES WITH IMMIGRATION

Zhumakulov Khurshijon,

KSPI, E-mail : xurshid81@gmail.com

Mirzaarapova Durdona,

master's student at NamSU

Abstract: The article considers nearly critical Galton -Watson branching processes with immigration and a functional limit theorem is proven for the fluctuation of a nearly critical branching process with immigration in the case where the average number of descendants of one particle slowly tends to unity from above .

Key words: nearly critical branching processes, functional limit theorem, conditional mathematical expectations, martingales, weak convergence.

In this paper we will consider sequences of Galton -Watson branching processes with immigration $X^{(n)} = \left\{ \mathrm{X}_k^{(n)},\ k \quad 0 \right\}$ n ∈N defined by the following recurrence relations

$$\mathrm{X}_0^{(n)} \quad 0,\ \mathrm{X}_k^{(n)} = \sum_{j=1}^{\mathrm{X}_{k-1}^{(n)}} \xi_{k,j}^{(n)} + \varepsilon_k^{(n)},\ n \quad N,$$

where for each $n \quad N$ set $\left\{ \xi_{k,j}^{(n)}, k, j \quad N \right\}$ and $\{ \varepsilon_k^{(n)}, k \quad N \}$ are sets of independent non-negative integer-valued and identically distributed random variables, independent of each other. Suppose that for each $n \quad N$ $E\left(\xi_{1,1}^{(n)}\right)^2 <$, and random variables $\varepsilon_k^{(n)}$ take only the values 1 and 0 with probabilities p_n and , $1 - p_n$ respectively $0 \quad p_n \quad 1$.

Let us introduce the following notations

$$m_n = E\xi_{1,1}^{(n)},\ \sigma_n^2 = \mathrm{var}\,\xi_{1,1}^{(n)}.$$

The process $X^{(n)}$ is called almost critical if $m_n \to 1$ at $n \to$.

Let us assume that the following conditions are met:

when $n \to$ the quantity m_n has an asymptotic representation

$$m_n = 1 + \alpha d_n^{-1} + o\left(d_n^{-1}\right), \qquad (5)$$

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 14 Issue 12, December 2024
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

where $\alpha > 0$, d_n is some sequence of positive numbers such that $d_n \to$ when $n \to$.

In works [5-9] the asymptotic behavior of the process is investigated $X^{(n)}$ in the case when the sequence nd_n^{-1} at $n \to$ has a finite limit.

Theorem 1. Let the following conditions be satisfied:

1) $m_n = 1 + \alpha d_n^{-1} + o(d_n^{-1})$ for $n \to$, $\alpha > 0$, d_n is a sequence of positive numbers such $d_n \to$ that $nd_n^{-1} \to$ for $n \to$,

2) $\lambda_n \to \lambda$, $\sigma_n^2 \to \sigma^2$ $d_n^{-1} b_n^2 \to b^2$ when $n \to$,

3) $m_n^{-2n} d_n^{-2} \operatorname{cov}\left(X_{n+m}^{(n)}, X_n^{(n)}\right) \to (2\alpha)^{-1}\left(b^2 + \alpha^{-1}\lambda\sigma^2\right)$ for $n \to$ all $m > 0$.

Then, when $n \to$ the random variable $W_n = \left(m_n^n d_n\right)^{-1} X_n^{(n)}$ converges in the mean square to some random variable W, and

$$EW = \alpha^{-1}\lambda,\ DW = (2\alpha)^{-1}\left(b^2 + \alpha^{-1}\lambda\sigma^2\right).$$

The following theorem covers the case where immigration into the population decreases as the series number increases.

Theorem 2. Let the following conditions be satisfied:

1) $m_n = 1 + \alpha d_n^{-1} + o(d_n^{-1})$ for $n \to$, $\alpha > 0$, d_n is a sequence of positive numbers such $d_n \to$ that $nd_n^{-1} \to$ for $n \to$,

2) $d_n \lambda_n \to \lambda$, $d_n \sigma_n^2 \to \sigma^2$ $d_n b_n^2 \to b^2$ when $n \to$,

3) $m_n^{-2n} \operatorname{cov}\left(X_{n+m}^{(n)}, X_n^{(n)}\right) \to (2\alpha)^{-1}\left(b^2 + \alpha^{-1}\lambda\sigma^2\right)$ for $n \to$ all $m > 0$.

Then, when $n \to$ the random variable $W_n = m_n^{-n} X_n^{(n)}$ converges in the mean square to some random variable W, and

$$EW = \alpha^{-1}\lambda,\ DW = (2\alpha)^{-1}\left(b^2 + \alpha^{-1}\lambda\sigma^2\right).$$

JOURNAL OF APPLIED SCIENCE AND SOCIAL SCIENCE
eISSN 2229-3113 pISSN 2229-3205
Volume 14 Issue 12, December 2024
Impact factor: 2019: 4.679 2020: 5.015 2021: 5.436, 2022: 5.242, 2023: 6.995, 2024 7.75

The aim of this paper is to study the asymptotic behavior of fluctuations $X_n(t)-EX_n(t)$ in the case when $m_n = 1+\alpha d_n^{-1} + o(d_n^{-1})$, where $\alpha > 0$, d_n is a sequence of positive numbers such that $d_n \to$ for $n \to$. In [54], a functional limit theorem for fluctuations is proved $n^{-\frac{1}{2}}\left(X_n(t)-EX_n(t)\right),\ t$ 0 in the case when $m_n = 1+\alpha n^{-1} + o(n^{-1})$. In [40], the asymptotic behavior of fluctuations is studied in the case when $m_n = 1+\alpha d_n^{-1} + o(d_n^{-1})$, where d_n is a sequence of positive numbers such that $nd_n^{-1} \to \beta <$ for $n \to$.

The following theorem defines the asymptotic behavior of the fluctuation process $X_n(t),\ t$ 0 in the case when $m_n = 1+2\alpha n^{-\frac{1}{2}} + o\ n^{-\frac{1}{2}}$, where $\alpha > 0$.

Theorem 3. Let the following conditions be satisfied:

1) $m_n = 1+2\alpha n^{-\frac{1}{2}} + o\ n^{-\frac{1}{2}}$ when $n \to$, $\alpha > 0$;

2) $\sigma_n^2 \to \sigma^2$ when $n \to$;

3) $n^{-1}\lambda_n \to \lambda,\ n^{-\frac{3}{2}} b_n^2 \to b^2$ when $n \to$;

4) $E\left(\xi_{1,1}^{(n)} - m_n\right)^2 I\left(\left|\xi_{1,1}^{(n)} - m_n\right| > \theta n\right) \to 0$ for $n \to$ any $\theta > 0$;

5) $n^{-\frac{3}{2}} E\left(\varepsilon_1^{(n)} - \lambda_n\right)^2 I\left(\left|\varepsilon_1^{(n)} - \lambda_n\right| > \theta n\right) \to 0$ for $n \to$ any $\theta > 0$.

Then weak convergence takes place

$$\left(m_n^{[nt]} n\right)^{-1}\left(X_n(t)-EX_n(t)\right) \to W\left(\left(2\alpha\right)^{-1}\left(\alpha^{-1}\lambda\sigma^2 + b^2\right)\right), \text{ at } n \to$$

in the space $D[0,T]$, where W is the standard Wiener process.

Literature

1. Жумакулов Х.К. Одна предельная теорема для ветвящихся процессов со стационарной иммиграцией. // Материалы Республиканской научной конференции “Современные проблемы математики, механики и информационных технологий”, 8 мая 2008. – Ташкент, 2008. –С. 88-89.

2. Жумакулов Х., Предельные теоремы для последовательности ветвящихся процессов с иммиграцией. Материалы научной конференции «Актуальные вопросы анализа» 22-23 апреля, 2016, Карши, стр. 327-328.

3. Жумакулов Х., Скорость роста почти критических ветвящихся процессов с иммиграцией. Тезисы докладов научной конференции «Проблемы современной топологии и её приложения» 5-6 мая, 2016, Ташкент, стр. 181-183.

4. Жумакулов Х. К., Каримова Ш. Я. К. НЕКОТОРЫЙ ЗАДАЧИ ВЕТВЯЩИХСЯ ПРОЦЕССОВ //Science and innovation. – 2024. – Т. 3. – №. Special Issue 50. – С. 28-33.

5. Жумакулов Х. К., Мирзаарапова Д. Р. К. ОБ АСИМПТОТИКЕ ПОЧТИ КРИТИЧЕСКОГО ВЕТВЯЩЕГОСЯ ПРОЦЕССА С ИММИГРАЦИЕЙ //Science and innovation. – 2024. – Т. 3. – №. Special Issue 50. – С. 113-119.

6. Хусанбаев Я.М. О поведении процесса Гальтона-Ватсона с иммиграцией. ДАН РУз, 2007, № 2, 3-5.

7. Хусанбаев Я.М. О флуктуации ветвящихся процессов с иммиграцией. УзМЖ, 2008, № 1, 112-126.

8. Ispany M., Pap G., Van Zuijlen M.C.A. Fluctuation limits of branching processes with immigration and estimation of the means. Adv.Appl. Probab., 2005, v. 37, 523-538.

9. Sriram T.N., Invalidtity of bootstrap for critical branching processes with immigration. Ann. Statist., 1994, v. 22, 1013-1023.