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GALTON-WATSON BRANCHING PROBABILITY FOR THE PROCESS INEQUALITIES

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Abstract: The rapid development of science and technology requires the creation of mathematical models for many processes. Even the mathematical model of reducing genetic diseases transmitted from generation to generation is a branching process. This work is devoted to the application of the theory of branching of random processes to some problems .

Key words: branching processes, random variables, generating function.

$Z_n,\ n=0,\ 1,\ 2,\ \ldots$ there p_k be a Galton-Watson critical process starting from a single particle, $f(x)=\underset{k=0}{\ }\ p_k x^k$ continuing with a single particle, k The probability of a particle. $B(x)=f\ (x),\ B=B(1)$ Let (B -second-order factorial moment) denote the radius of convergence of the series by R. The $f(x)$ following is valid .

Theorem 1. $B>0,\ R>1$ and $0<y_0<R-1$ in that case

$$P\left(Z_n \quad k\right)<\left(1+y_0\right)\ 1+\frac{1}{B_0\ n/2+1/y_0}^{\ -k} \qquad (1)$$

here $B_0=B(1+y_0)$

Proof of Theorem 1. $y_n,\ n=0,\ 1,\ 2,\ \ldots$ sequence $0<y_0<R-1$ at $y_{n-1}=y_n+\frac{B_0}{2}y_n^2$ let the recurrence formula be satisfied. y_n the sequence decreases. Therefore

$$\frac{1}{y_n}=\frac{y_{n-1}}{y_{n-1}\ y_n}=\frac{y_n+\frac{B_0}{2}y_n^2}{y_{n-1}\ y_n}=\frac{1}{y_{n-1}}+\frac{B_0y_n}{2y_{n-1}}<\frac{1}{y_{n-1}}+\frac{B_0}{2}$$

calculating the sum

$$\frac{1}{y_n}<\frac{1}{y_0}+\frac{B_0}{2}n \qquad (2)$$

we get. $B(x)$ If we assume that the function is increasing

$$f(x)=1+\frac{f\ (1)}{1!}(x-1)+\frac{f\ (1)}{2!}(x-1)^2+\frac{f\ (1)}{3!}(x-1)^3+\ldots=$$

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$$=1+(y_n+1-1)+\frac{(y_n+1-1)^2}{2}\quad f\ (1)+\frac{f\ \ (1)}{3}y_n+...$$

$$1+y_n+\frac{B(1+y_n)}{2}y_n^2<1+y_n+\frac{B_0}{2}y_n^2=1+y_{n-1}.$$

As a result

$$f(1+y_n)<1+y_{n-1},$$

$$f_2(1+y_n)=f(f(1+y_n))<f(1+y_{n-1})<1+y_{n-2},$$

$$f_3(1+y_n)<f_2(1+y_{n-1})<f(1+y_{n-2})<1+y_{n-3},$$

………

$$f_n(1+y_n)<1+y_0, \qquad (3)$$

Here $f_n-n\quad f$ The n interaction of and f_k the function is exponentially increasing. $x>1$ Let it be. In that case

$$P(Z_n\quad k)\quad x^{-k}f_n(x). \qquad (4)$$

It is reasonable, now (4) is enough $x=1+y_n$, considering that

$$P(Z_n\quad k)<(1+y_n)^{-k}f_n(1+y_n)<(1+y_0)\quad 1+\frac{1}{B_0\, n/2+1/y_0}\quad{}^{-k}$$

the theorem has been proven.

Let's look at examples of theorem 1.

Example 1 . $k=\sqrt{n},\ y_0=\frac{1}{\sqrt{n}}$ In that case, the right-hand side of inequality (1) is as follows:

$$1+\frac{1}{\sqrt{n}}\quad 1+\frac{1}{\frac{B_0 n}{2}+\sqrt{n}}\quad{}^{-n}\sim e^{-\frac{2}{B_0\sqrt{n}}}\to 1$$

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so for large enough n, $P\left(Z_n \quad \sqrt{n}\right) \quad 1$ and theorem 1 will not have any useful information in this case.

Using Kolmogorov 's hunting theorem

$$P\left(Z_n \quad \sqrt{n}\right) \quad P\left(Z_n \quad 0\right) \sim \frac{2}{nB}$$

we will have .

Example 2. $y_0 = \frac{1}{n}, \ k = n \ln n$ In this case, the right-hand side of equation (1) is

$$1+\frac{1}{n} \quad 1+\frac{1}{\frac{B_0 n}{2}+n}^{\ n\ln n} \sim 1+\frac{1}{n\ \frac{B_0}{2}+1}^{\ n\ \frac{B_0}{2}+1\ \frac{n\ln n}{n\ \frac{B_0}{2}+1}} \sim e^{-\frac{\ln n}{n\ \frac{B_0}{2}+1}} \to 0$$

So in this case, $P\left(Z_n \quad k\right)$ the inequality with the exponent tends to zero.

Literature

1. Жумакулов Х. К., Каримова Ш. Я. К. НЕКОТОРЫЙ ЗАДАЧИ ВЕТВЯЩИХСЯ ПРОЦЕССОВ //Science and innovation. – 2024. – Т. 3. – №. Special Issue 50. – С. 28-33.
2. Жумакулов Х. К., Мирзаарапова Д. Р. К. ОБ АСИМПТОТИКЕ ПОЧТИ КРИТИЧЕСКОГО ВЕТВЯЩЕГОСЯ ПРОЦЕССА С ИММИГРАЦИЕЙ //Science and innovation. – 2024. – Т. 3. – №. Special Issue 50. – С. 113-119.
3. Хусанбаев Я. М., Жумакулов Х. К. О сходимости почти критических ветвящихся процессов с иммиграцией к детерминированному процессу //O 'ZBEKISTON MATEMATIKA JURNALI. – С. 142.
4. А. В. Нагаев, Большие уклонения для одного класса распределений, в сб.: «Предельные теоремы», Ташкент, 1963, 56—68.
5. С. В. Нагаев, Интегральная предельная теорема для больших уклонений,- ДАН СССР, 148, 2 (1963), 280.