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CONVEXITIES, INFLECTION POINTS, AND ASYMPTOTES OF THE FUNCTION. FULL FUNCTION CHECK

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Abstract: Convexity, inflection points, and asymptotes of a function in this paper. The full function check is thought out and explained with examples.

Keywords: function, skewness, point, derivative, graph.

Suppose that $f(x)$the function $(a,b)$is given by , and let be $x_1, x_2 \in (a,b)$for $x_1 < x_2$.

$f(x)$ a straight line passing through the points $y = l(x)$ of the graph of a function $(x_1, f(x_1)), (x_2, f(x_2))$, it is as follows:

$$l(x) = \frac{x_2 - x}{x_2 - x_1} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2)$$

will be.

Definition **1. If** in $(x_1, x_2) \subset (a,b)$any interval settled $\forall x \in (x_1, x_2)$for

$$f(x) \leq l(x)(f(x) < l(x))$$

If , $f(x)$the function $(a,b)$is called a concave (strictly concave) function.

Definition 2. If on $(x_1, x_2) \subset (a,b)$any island for settling $\forall x \in (x_1, x_2)$down

$$f(x) \geq l(x)(f(x) > l(x))$$

If , $f(x)$ the function (a,b) is called a convex (strictly convex) function.

concave and convex functions are depicted in Figure 1 :

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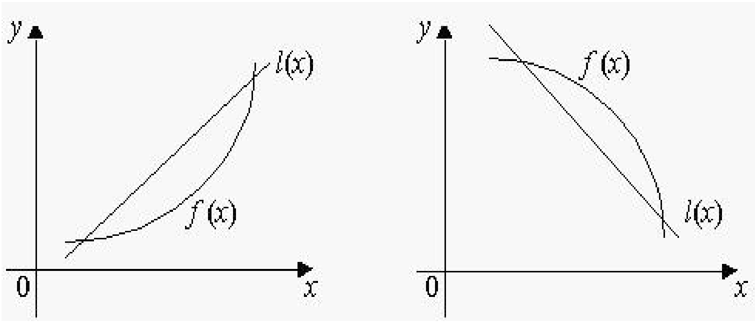

Drawing 1.

Let's say, $\alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_1 + \alpha_1 = 1$be and $\forall x_1, x_2 \in (a, b)$be. The concave and convexity of a function can also be defined as follows.

Definition 3. If

$$f(\alpha_{1.}x_1 + \alpha_2 x_2) \leq \alpha_1 f_1(x_1) + \alpha_2 f(x_2)$$

$$(f(\alpha_{1.}x_1 + \alpha_2 x_2) < \alpha_1 f(x_1) + \alpha_2 f(x_2))$$

If , $f(x)$the function $(a, b)$is said to be concave (strictly concave).

Definition 4. If

$$f(\alpha_{1.}x_1 + \alpha_2 x_2) \geq \alpha_1 f_1(x_1) + \alpha_2 f(x_2)$$

$$(f(\alpha_{1.}x_1 + \alpha_2 x_2) > \alpha_1 f(x_1) + \alpha_2 f(x_2))$$

If , $f(x)$the function is said to (a, b)be convex (strictly convex) .

Example 1. This

$$f(x) = x^2$$

The function is R also a strictly concave function.

Solution . Using Definition 3, we find:

$$f(\alpha_1 x_1 + \alpha_2 x_2) = (\alpha_1 x_1 + \alpha_2 x_2)^2 = (\alpha_1 x_1)^2 + 2\alpha_1 \alpha_2 x_1 x_2 + (\alpha_2 x_2)^2 <$$

$$< \alpha_1^2 x_1^2 + \alpha_1 \alpha_2 (x_1 + x_2)^2 + \alpha_2^2 x_2^2 = \alpha_1 x_1^2 (\alpha_1 + \alpha_2) + \alpha_2 x_2^2 (\alpha_1 + \alpha_2) =$$

$$= \alpha_1 x_1^2 + \alpha_2 x_2^2 = \alpha_1 f(x_1) + \alpha_2 f(x_2)$$

Theorem 1. Suppose that $f(x)$the function is (a, b)given at **and** has a derivative at . $f(x)$For the function $f^{'}(x)$ to (a, b) **be** concave at (strictly concave at) , $f^{'}(x)$ **then the** function is (a, b)increasing at (strong growth) is necessary and is diverse .

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Necessity. Let $f(x)$the function (a,b)be concave . In $\forall x_1, x_2 \in (a,b)$,that case $x_1 < x_2$, $\forall x \in (x_1, x_2)$for

$$f(x) \leq \frac{x_2 - x}{x_2 - x_1} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2)$$

from which

$$\frac{f(x) - f(x_1)}{x - x_1} \leq \frac{f(x_2) - f(x)}{x_2 - x}$$

It turns out that ($(x_2 - x_1) = (x_2 - x) + (x - x_1)$ it was said). In the next inequality, $x \to x_1$after $x \to x_2$passing to the limit,

$$f'(x_1) \leq \frac{f(x_2) - f(x_1)}{x_2 - x_1},$$

$$f'(x_2) \geq \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

We find that it will be. From it $f'(x_1) \leq f'(x_2)$ to be come It turns out that $f'(x)$ the function is increasing on (a,b) .

$f(x)$the function (a,b)be strictly concave. In that case

$$\frac{f(x) - f(x_1)}{x - x_1} < \frac{f(x_2) - f(x)}{x_2 - x}$$

According to Lagrange's theorem,

$$\frac{f(x) - f(x_1)}{x - x_1} = f'(c_1), x_1 < c_1 < x;$$

$$\frac{f(x_2) - f(x)}{x_2 - x} = f'(c_2), x < c_2 < x_2$$

It is, and it follows from it $f\ (x_1) < f\ (x_2)$ that it is .

Sufficiency . Let $f'(x)$ the function (a,b) be increasing (strictly increasing): $\forall x_1, x_2 \in (a,b)$, $x_1 < x_2$ at

$f'(x_1) \leq f'(x_2) (f'(x_1) < f'(x_2))$.

Using Lagrange 's theorem , we find :

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$$\frac{f(x)-f(x_1)}{x-x_1}=f'(c_1), \quad x_1<c_1<x;$$

$\frac{f(x_2)-f(x)}{x_2-x}=f'(c_2), \quad x<c_2<x_2.$

Ravshanki ,. $x_1<c_1<x<c_2<x_2 \Rightarrow c_1<c_2$ D e mak , $f'(c_1)\le f'(c_2)$ $(f'(c_1)<f'(c_2))$ above from these days

$\frac{f(x)-f(x_1)}{x-x_1}\le\frac{f(x_2)-f(x)}{x_2-x}$ $\left(\frac{f(x)-f(x_1)}{x-x_1}<\frac{f(x_2)-f(x)}{x_2-x}\right)$

to be come It comes out . This and $f(x)$ function (a,b) at deep (solid) that it is a swamp indicates .

Huddy to that oh 'x shash , the following theorem also It is proven .

Theorem 2 . $f(x)$ function (a,b) at given to be , then $f'(x)$ to the end has Let it be .

$f(x)$ function (a,b) at convex (solid) convex for $f'(x)$ of (a,b) at decreasing (constant) decreasing (to be) necessary and it is different .

Theorem 3 . $f(x)$ function (a,b) in the interval to be concave (convex) for (a,b) at

$$f''(x)\ge 0 \quad (f''(x)\le 0)$$

It is necessary and sufficient to be.

The proof of this theorem follows from the above theorem and the theorem on the monotonicity of a function.

Example 2. This

$$f(x)=ln\,x\,(x>0)$$

the function is convex.

For this function

$$f''(x)=-\frac{1}{x^2}<0$$

According to Theorem 2, the given $f(x)=\ln x$ function $(0,+\quad)$ is also strictly convex.

Inflection points of a function. Suppose that $f(x)$the function $X\subset R$is given on a set, and $x_0\in X,(x_0-\delta,x_0+\delta)\subset X,\delta>0$let .

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Definition 5. If $f(x)$ a function is $(x_0-\delta, x_0)$ concave (convex) at $(x_0, x_0+\delta)$ and convex (concave) at, $x_0$the point is called $f(x)$*an inflection point* of the function .

Suppose $f(x)$ the function $(x_0-\delta, x_0+\delta)$ has a derivative at . $f''(x)$ If $\forall x \in (x_0-\delta, x_0)atf''(x) \geq 0$ $(f\ (x)\quad 0)$,

$\forall x \in (x_0, x_0+\delta,)atf''(x) \leq 0$ $(f\ (x)\quad 0)$,

If , $f'(x)$the function $x_0$reaches an extremum at the point and therefore , $f''(x_0)=0$becomes . Therefore ,$f(x)$ function bending at the point $f''(x)=0$ It will be .

Example 3 . This $f(x)=x^3$ function $x_0=0$ on point bends .

This function for

$$f''(x)=6x$$

is,

$\forall x \in (-\delta, 0)atf''(x)<0$

$\forall x \in (0, \delta,)atf''(x)>0$ $(\delta>0)$

will be.

Asymptotes of a function graph

Suppose that $f(x)$the function $X \subset R$is given on a set, and $x_0$the point $X$is a limit point of the set.

Definition 6. If this

$$\lim_{x\to x_0+0} f(x), \lim_{x\to x_0-0} f(x)$$

if one or both of the limits are infinite, $x=x_0$A straight line $f(x)$is called the vertical asymptote of the graph of a function.

For example ,$f(x)=\frac{1}{x}$ function graph for $x=0$ right line vertical asymptote It will be .

Let's say $f(x)$the function $(x_0, +\infty)$is defined in .

Definition 7. If such $k$and $b$such numbers are found,

$$f(x)=kx+b+\alpha(x)(x\to+\infty\ \ da\ \ \alpha(x)\to 0)$$

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If , then $y = kx + b$ the straight line $f(x)$ is called the oblique asymptote of the graph of the function.

Theorem 4. so that $f(x)$the graph of the function $y = kx + b$has a g - axis asymptote

$$\lim_{x\to+\infty} \frac{f(x)}{x} = k,\ \lim_{x\to+\infty} (f(x) - kx) = b$$

It is necessary and necessary .

Necessity. $y = kx + b$straight line $f(x)$function Let the graph of ω be an asymptote . Then

$$f(x) = kx + b + \alpha(x)$$

is, $x \to +\infty$да$\alpha(x) \to 0$is. Taking this equality into account, we find:

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{kx + b + \alpha x}{x} = k;$$

$$\lim_{x\to+\infty} (f(x) - kx) = \lim_{x\to+\infty} (b + \alpha(x)) = b.$$

Sufficiency . This

$$\lim_{x\to+\infty} \frac{f(x)}{x} = k,\ \lim_{x\to+\infty} (f(x) - kx) = b$$

relationships should be appropriate. From these relationships

$$(f(x) - kx) - b = \alpha(x) \to 0 \Rightarrow f(x) = kx + b + \alpha(x)$$

It turns out to be.

Example 5 . $f(x) = \frac{x^3}{(x-1)^2}$Find the oblique asymptote of the function.

For this function

$$k = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x^2}{(x-1)^2} = 1;$$

$$b = \lim_{x\to+\infty} (f(x) - kx) = \lim_{x\to+\infty} \left(\frac{x^3}{(x-1)^2} - x\right) = 2$$

Thus, $y = x + 2$a straight line is a diagonal asymptote of the graph of a given function.

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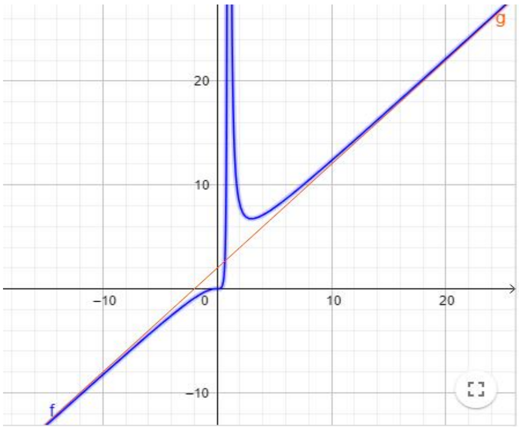

Fully investigate and graph the function

$y = f(x)$It is advisable to study the function in a certain sequence.

1. Find the domain of the function.

2. Find the points of intersection of the graph with the coordinate axes (if possible).

3. Find the intervals on which the sign of the function does not change ($f(x) > 0$ or $f(x) < 0$).

4. Determine whether the function is even, odd, or neither even nor odd.

5. Find the asymptotes of the graph of the function.

6. Find the intervals of monotonicity of the function.

7. Find the extrema of the function.

8. Find the convex intervals and inflection points of the graph of the function.

Example 6. This

$y = \frac{1}{x} + 4x^2$Examine the function and graph it.

Solution: 1) domain of definition of the function$D(f)$: $(-\infty; 0) \cup (0; +\infty)$; $x = 0$breakpoint of a function;

2) We find the points of intersection of the function with the coordinate axes:

$Ox$the axis $y = 0, x = -\frac{\sqrt[3]{2}}{2}$, $Oy$the axis .

3) $\left(-\infty; -\frac{\sqrt[3]{2}}{2}\right) \cup (0; +\infty)$ da $y > 0$,$\left(-\frac{\sqrt[3]{2}}{2}; 0\right)$ da $y < 0$;

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4) $f(-x) =-\frac{1}{x}+4x^2 \neq f(x) \neq f(-x)$the function is neither even nor odd;

5) Vertical asymptote $x = 0$.We find the oblique asymptote.$y = kx + b$

6) $\lim\limits_{x\to\infty}\frac{\frac{1}{x}+4x^2}{x} = \lim\limits_{x\to\infty}\left(\frac{1}{x^2}+4x\right) = \infty$there is no obtuse asymptote;

7) We find the monotone intervals and extremum values of the function:

$y' = \left(\frac{1}{x}+4x^2\right)' =-\frac{1}{x^2}+8x = \frac{-1+8x^3}{x^2}$;$y' = 0 \Rightarrow -1+8x^3 = 0 \Rightarrow$

$x = \frac{1}{2}$ −extremum point.

x	$(-\infty;0)$	$(0;\frac{1}{2})$	$\frac{1}{2}$	$\left(\frac{1}{2};+\infty\right)$
y	↘	↘	$y_{min} = 3$	↗
y'	$-$	$-$	0	$+$

8) We find convex intervals and inflection points:

$$y'' = \left(-\frac{1}{x^2}+8x\right)' = \frac{2}{x^3}+8 = \frac{2(4x^3+1)}{x^3}; y'' = 0$$

$\Rightarrow 4x^3+1 = 0 \Rightarrow x =-\frac{\sqrt[3]{2}}{2}$ −inflection point.

x	$\left(-\infty;-\frac{\sqrt[3]{2}}{2}\right)$	$-\frac{\sqrt[3]{2}}{2}$	$\left(-\frac{\sqrt[3]{2}}{2};0\right)$	$(0;+\infty)$
y	∪	$y = 0$	∩	∪
y''	$+$	0	$-$	$+$

Let's graph the function using the above findings:

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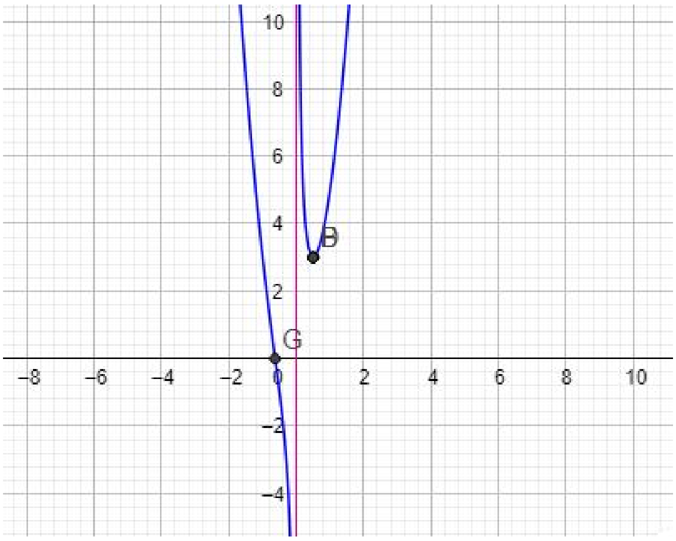

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