

## THE JACOBIAN METHOD FOR BRINGING A QUADRATIC FORM TO CANONICAL FORM

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**Abstract:** This article extensively examines the problem of transforming a quadratic form into its canonical form. There are several methods to simplify and bring a quadratic form into a convenient and straightforward form, one of which is the Jacobian method. The article discusses how to apply the Jacobian method, its advantages, and provides examples and practical instructions related to the calculation processes. This method pays special attention to the coefficients and signs of the quadratic form.

**Keywords:** Quadratic form, Jacobian method, principal minor, coefficient, matrix, basis.

**Introduction.** A quadratic form is a mathematical expression that includes second-degree terms. Quadratic forms are widely used in solving various problems in mathematics and physics. The general form of a quadratic expression is represented as follows:

$$f = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

in this case  $a_{11}, a_{22}, a_{33}, a_{12} = a_{21}, a_{13} = a_{31}, a_{23} = a_{32}$  the coefficients of the quadratic form are calculated. After studying problems such as simplifying the quadratic form, determining whether its sign is positive or negative, and similar issues, we will then study the Jacobian method. Initially, we find the number of positive  $\Delta_{i-1}$  and  $\Delta_i$  negative coefficients of the quadratic form.

That is, if the signs of the  $\xi_i^2$  coefficients are the same for both, the expression will have a positive coefficient; otherwise, it will have a negative coefficient. This is related to the number of negative coefficients in front of the squares.

$$1, \Delta_1, \Delta_2, \Delta_3, \dots, \Delta_n$$

this means that it is equal to the number of sign changes in the sequence.

In the special case where it is true  $\Delta_1 > 0, \Delta_2 > 0, \Delta_3 > 0, \dots, \Delta_n > 0$ , the canonical form of the quadratic form will be

$$A(x, x) = \lambda_1 \xi_1^2 + \lambda_2 \xi_2^2 + \dots + \lambda_n \xi_n^2$$

it will be like this, and  $\lambda_i > 0$  it will be. This means  $A(x, x) = 0$  that for any value of  $x$ , it will be  $\xi_1 = \xi_2 = \dots = \xi_n$  true, and it will only be true  $A(x, x) = 0$  when it is. We now compute the following principal minors of this matrix:

$$\Delta_1 = a_{11}; \quad \Delta_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}; \quad \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \dots & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & \dots & a_{nn} \end{vmatrix}$$

**Theorem:** Let,  $A(x, x)$  the matrix  $A = a_{ij}$  of the quadratic form  $f_1, f_2, f_3, \dots, f_n$  be in the basis. If  $\Delta_1 > 0, \Delta_2 > 0, \Delta_3 > 0, \dots, \Delta_n > 0$  are non-zero, then there exists a basis in  $e_1, e_2, \dots, e_n$  which the form is transformed into its canonical form, and its canonical  $A(x, x)$  form will be as follows:

$$A(x, x) = \frac{1}{\Delta_1} \xi_1^2 + \frac{\Delta_1}{\Delta_2} \xi_2^2 + \frac{\Delta_2}{\Delta_3} \xi_3^2 + \dots + \frac{\Delta_{n-1}}{\Delta_n} \xi_n^2.$$

This theorem is also called the Jacobian theorem, and now we will see its application through an example.

**Example.** Transform the following quadratic form into its canonical form:

$$f = x_1^2 + 17x_2^2 + 3x_3^2 + 4x_1x_2 - 2x_1x_3 - 14x_2x_3$$

**Solution.** We will construct the matrix of the quadratic form in the basis, and it will be as follows:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 17 & -7 \\ -1 & -7 & 3 \end{pmatrix}$$

it will be in the following form. Now, we will calculate the principal minors of this matrix:

$$\Delta_1 = 1 \quad 0; \Delta_2 = \begin{vmatrix} 1 & 2 \\ 2 & 17 \end{vmatrix} = 17 - 4 = 13 \quad 0; \Delta_n = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 17 & -7 \\ -1 & -7 & 3 \end{vmatrix} = 1 \quad 0.$$

Now, according to the theorem, the canonical form of the quadratic form will be as follows:

$$f = \frac{1}{\Delta_1} \xi_1^2 + \frac{\Delta_1}{\Delta_2} \xi_2^2 + \frac{\Delta_2}{\Delta_3} \xi_3^2 = \xi_1^2 + \frac{1}{13} \xi_2^2 + 13 \xi_3^2$$

It will be equal to.

**Conclusion.** The Jacobian method is an effective tool for bringing quadratic forms into canonical form. This method allows for a deeper study of the properties of quadratic forms and simplifies calculations. The Jacobian theorem helps analyze the positivity or negativity of a quadratic form and obtain its simplified form. Quadratic forms and their analysis are widely used in mathematics, so learning such methods is important not only in theory but also in practical problems.

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